DISTRIBUTION FUNCTION FOR CONVECTIVE THERMALS IN THE ATMOSPHERIC BOUNDARY LAYER

ALEXANDER N. VUL'FSON

Institute of Oil and Gas Problem, Russian Academy of Sciences, Russia
E-mail: vulfson@ogri.ru

The problem of statistical size distribution for an ensemble of thermals taking into account the multilayer structure of turbulence in the atmospheric convective boundary layer are considered. A special form of the size distribution function of convective thermals in the atmospheric boundary layer is derived from similarity theory and Boltzmann statistics. The expression for the distribution density of convective plums in the surface convective sublayer takes the form

$$\frac{N_r}{N_0} = 1.56\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{5/3} exp\left\{-\frac{5}{3}\left(\frac{r}{r_0}\right)\right\},$$

where r – radius of thermal; r_0 – the most probably radius corresponding to this distribution; $N_r dr$ - the number of convective elements in a unit area whose radii are between r and $r + dr$; N_0 - the total number of convective elements in a unit area.

Obviously that distribution for small r has Oboukhov's spectrum asymptotic. According to the multilayer structure of the convective layer, this relation changes in passing from the surface sublayer to the mixed sublayer. The results obtained are shown to agree well with known experimental data on size distributions of convective elements.

NON-TRADITIONAL EDDY DYNAMICS

ACHIM WIRTH, JACQUES VERRON

Laboratoire des Ecoulements Géophysiques et Industriels CNRS, France
E-mail: achim.wirth@hmg.inpg.fr

When considering the eddy dynamics in a rotating frame the rotation vector is usually assumed to be collinear to gravity and the horizontal component of the rotation vector is neglected (traditional approximation).

We present numerical results of the geostrophic adjustment problem and the interaction of two eddies in which the traditional approximation is relaxed. A special interest is put on the evolution and structure of the vertical velocity.

The numerical model used (HAROMOD) solves the three-dimensional Navier-Stokes equations subject to the Boussinesq approximation, a free-slip boundary condition on the top and an no-slip boundary condition at the bottom. The results are compared to integrations performed with a hydrostatic ocean model (OPA).

SPATIAL DYNAMICS OF SOME CLASSICAL OPEN SHEAR FLOWS

XILIN XIE[1], WEIWEI MA[2], HUILIANG ZHOU[1]

[1]Department of Mechanics & Engineering Science, Fudan University, China
[2]College of Basic Science, Donghua University, China
E-mail: xiexilin@fudan.edu.cn

Summary

Real open shear flows are abundant in large scale vortical structures those dynamical behaviors dominate the spatial evolutions of open flows. On the other hand, vortical structures with their dynamical behaviors can be considered as the properties of the Navier-Stokes equation (NSE). Therefore, it is significant to unit mathematical and experimental studies. In this study, the elementary efforts have been paid under this way.

Foundations of experimental studies

Study on Global Dynamics

The so-called linear and nonlinear transfer functions, denoted by L_m and Q^m_{ij}, in poly-spectrum analysis are determined trough the linear system:

$$\left\{ \begin{array}{rclcl} \langle Y_m X^*_m \rangle & = & L_m \langle |X_m|^2 \rangle & + & Q^m_{ij} \langle X_m X^*_i X^*_j \rangle^* \\ \langle Y_m X^*_k X^*_l \rangle & = & L_m \langle X_m X^*_k X^*_l \rangle & + & Q^m_{ij} \langle X_i X_j X^*_k X^*_l \rangle \end{array} \right. \tag{1}$$

where $f_i + f_j = f_k + f_l = f_m$. Subsequently, the energy relation:

$$\langle |Y_m|^2 \rangle = \underbrace{|L_m|^2 \langle |X_m|^2 \rangle}_{S^I_L(f_m)} + \underbrace{2\,\mathrm{Re}[L_m Q^{m*}_{ij} \langle X_m X^*_i X^*_j \rangle]}_{S^I_{LQ}(f_m)} + \underbrace{Q^m_{ij}\, Q^{m*}_{kl} \langle X_i X_j X^*_k X^*_l \rangle}_{S^I_Q(f_m)} \tag{2}$$

where $S^I_L(f_m)$, $S^I_Q(f_m)$ and $S^I_{LQ}(f_m)$ are defined as *linear*, *nonlinear* and *linear-nonlinear mechanisms* respectively, can be constructed based on the fundamental relation, i.e., $Y_m = L_m X_m + Q^m_{ij} X_i X_j$. This relation is originated from the local stability analysis so that the poly-spectrum analysis is in the local sense. However, the linear system (1) can be extended to two arbitrary points $\mathbf{r_0}$ and $\mathbf{r}$ in a flow field to determine the corresponding transfer functions $L_m(\mathbf{r}; \mathbf{r_0})$ and $Q^m_{ij}(\mathbf{r}; \mathbf{r_0})$ since the coefficient matrix of (1) is Hermitian. Furthermore, as we want to mention, the relation (2) is still held on by almost frequencies, particularly the dominant ones. The validity of the relations (1) and (2) between arbitrary $\mathbf{r_0}$ and $\mathbf{r}$ as indicated by experiments on some classical open shear flows implies that *the fundamental relation in poly-spectrum analysis is still sustained in the global sense*. In mathematics, we can found a kind of transfer functions those corresponding fundamental relation is equivalent to the general NSE in the temporal Fourier Space. On the other hand, the phase-angle of cross-spectrum $S_{yx}(f_m) = \langle Y_m X^*_m \rangle$ is adopted to detect the spatial phase pattern.

Study on Local Dynamics

According to the hydrodynamic stability analysis, the so-called self-bispectrum, denoted by $S_{yyy}(f_m; f_i, f_j) = \langle Y_m Y^*_i Y^*_j \rangle$, in poly spectrum analysis can be considered as the measure of the contribution due to the nonlinearity of NSE to the rate of local spatial evolution of the frequency f_m.

Experimental results

The global spatial energy relation (2) and self-bispectrum have been used to study the spatial dynamics of axisymmetric and mirror-symmetric shear flows, variable density round jets and wake flows. Some general properties can be concluded as follows.

1. *Energy Transfer Mechanism*: $S^I_{LQ}(f_f) \approx -S^I_{LQ}(f_{f/2})$ in pairing-merging process, where f_f and $f_{f/2}$ denote the fundamental frequency with its first subharmonic, as $\mathbf{r_0}$ and $\mathbf{r}$ located in roll-up and pairing-merging regimes respectively.

2. *Energy Resonance Mechanism*: $S^I_L(f_G) + S^I_Q(f_G) \approx -S^I_{LQ}(f_G)$ with $S^I_L(f_G)$, $S^I_Q(f_G)$ and $|S^I_{LQ}(f_G)| \gg S_{yy}(f_G)$, where f_G denotes the global oscillation frequency, as $\mathbf{r_0}$ and $\mathbf{r}$ are both located in the oscillation regime.

3. In any open flow, when $\mathbf{r_0}$ is fixed in the upstream ordered regime but $\mathbf{r}$ is moving downstream more and more, the nonlinear mechanisms for almost frequencies as compared to the corresponding linear and linear-nonlinear ones will play the dominant role finally. This process is corresponding to the 'lost of spatial phase relation', i.e., the spatial phase relation between $\mathbf{r_0}$ and $\mathbf{r}$ detected through the phase of cross-spectrum is evolved into disordered status. In the local sense, the nonlinearity of NSE plays the important role in turbulence generation for any open flow as indicated by self-bispectrum.

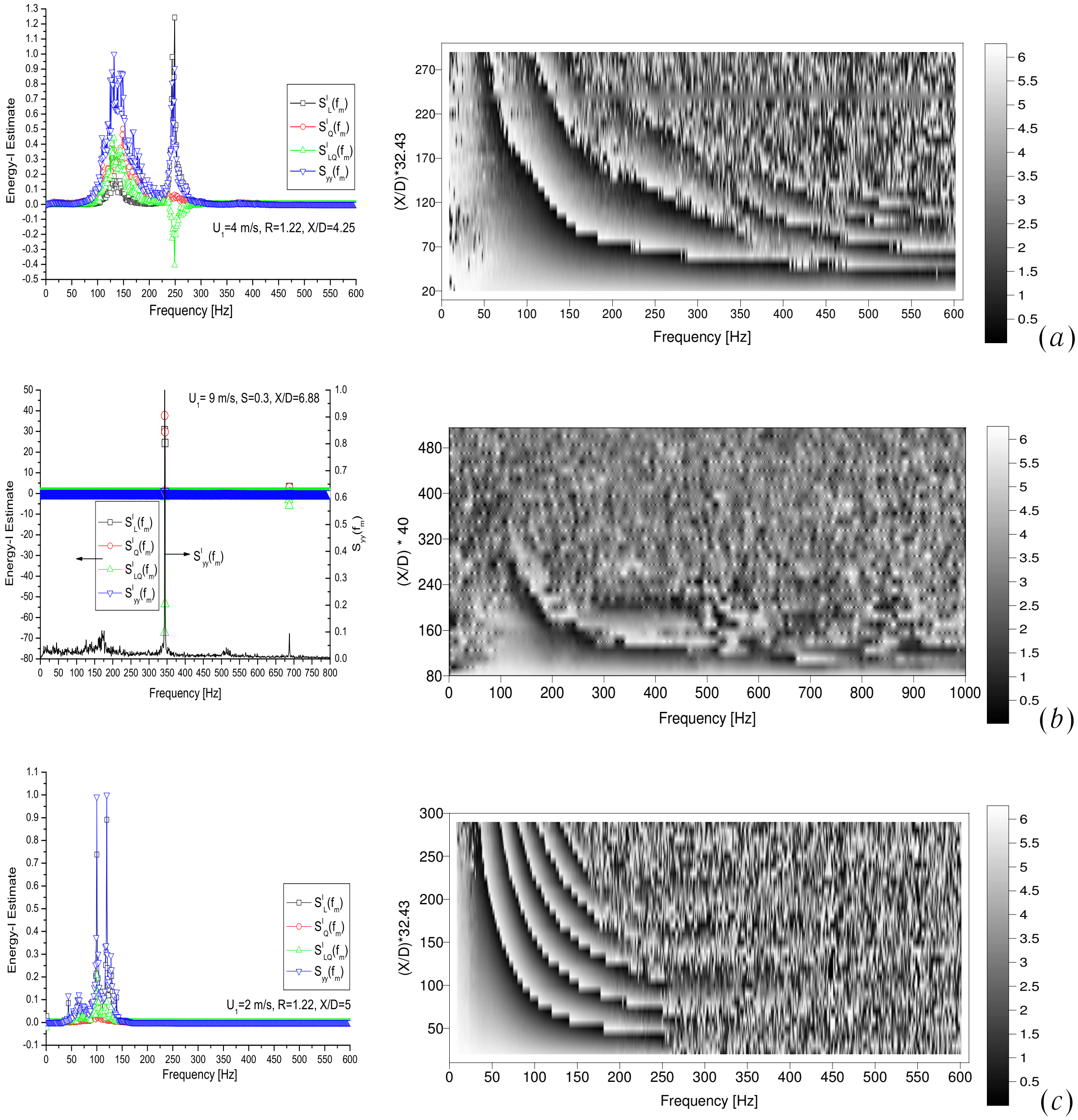

Fig 1. Comparing the energy relation of the first kind with the spatial phase pattern corresponding to a certain large scale structure in a certain flow as indicated in Table.1: (a) $\mathcal{S}1$ *energy transfer* from the fundamental to its subharmonic through S^I_{LQ} with ordered phase pattern; (b) $\mathcal{S}2$ *energy resonance* between S^I_L, S^I_Q and S^I_{LQ} on global oscillation frequency with disordered phase pattern; (c) $\mathcal{S}3$ linearity of helical structure that corresponds to 2-torus in phase space with quite ordered phase pattern.

References

[1] Hajj M. R.; Miksad R. W.; Power E. J. Subharmonic Growth by Parametric Resonance. *J. Fluid Mech.* **236** (1992), pp. 385–413.

[2] Xie X. L.; Ma W. W.; Zhou H. L. Bifurcations, transitions and attractors of real open flows. To be published in *Journal of Mechanical Engineering Science, I Mech. E*, **220** (2006).

Vortical Structures	Span of Dynamics	Governing Mechanisms	Phase Patterns	Flow Fields
$\mathcal{S}1$ Axisymmetric	Roll-up $\Rightarrow$ Merging	Linear $\Rightarrow$ Nonlinear	Ordered	Axisymmetric Shear Flow
$\mathcal{S}2$ Axisymmetric	Global Oscillation	All Mechanisms	Disorde-red	Variable-Density Jet
$\mathcal{S}3$ Helical	2-torus Regime	Linear	Ordered	Axisymmetric Shear Flow
$\mathcal{S}4$ Spanwise	Pairing $\Rightarrow$ Merging	Linear & Nonlinear		Mirror-symmetric Shear Flow

Table 1. Comparison of global dynamics of different large-scale structures in different open shear flows. The ordered status of phase patterns can be considered as the measure of the validity of linear hydrodynamic stability. As indicated, helical structure that is dominated by linear mechanism corresponds to quite ordered phase pattern, however, global self-excited oscillation of axisymmetric structure controlled by linear and nonlinear mechanisms corresponds to disordered pattern.

VORTEX EQUILIBRIA IN CONFINED DOMAINS

LUCA ZANNETTI

Dipartimento di Ingegneria Aeronautica e Spaziale, Politecnico di Torino, Italy
E-mail: luca.zannetti@polito.it

Point vortices and vortex patches are widely used to model flow regions with closed streamlines. The two models can be connected. Point vortices can in fact be considered as vanishing area vortex regions and vortex patches can be obtained as their accretions. The growing process generates a family of regions with closed streamlines.

The present work is aimed at studying families of regions whose elements are patches with the same circulation as the nascent point vortex region and which are embedded in a potential flow with closed streamlines. Thus, the families here considered are formed by two-level piecewise constant vorticity regions. The vorticity is $\omega = 0$ in the outer part and $\omega = \kappa/A_\omega$ in the inner part, with κ being the circulation of the original point vortex and A_ω the area of the inner patch. The point vortex is the extremum element defined by $A_\omega = 0$. The other extremum is the vortex patch that fills the entire region with closed streamlines.

It can be shown that the vortex patch model has physical relevance in the modelling of finite area separated flow regions. For a proper choice of the jump of the Bernoulli constant, with respect to the external flow, the vortex patch can be considered as the limit solutions of the Navier-Stokes equations for the Reynolds number going to infinity. Thus, the connection between vortex patches and point vortices has practical importance. In fact, if a standing vortex solution does not exist in a flow past a body, it could be conjectured that the entire family of growing vortex patches does not exist and, as a consequence, a finite area separated flow region does not exist either.

For instance, in the flow past a semicircular bump, there is a locus (the Föppl curve) of possible standing single point vortices. In [1] it is shown that, for each standing vortex of this flow, there is a family of vortex patches that goes from the zero area point vortex to a maximum area vortex region that is bounded by the solid body. In [2] the Föppl curve concept has been generalized by showing that a locus of standing vortices can be found in any bounded simply connected domain.

When the solid wall that confines the flow domain has a sharp edge, the flow has to separate at the edge and the number of possible standing vortices reduces to a finite or null number. In [2] it is shown that the existence, or non existence, of standing vortices relevant to flow separating at a wall singularity depends on the nature of the singularity. The present work is aimed at showing, at least for some specific wall geometries, that the non existence of a standing point vortex solution does not allow for the existence of the entire family of vortex patches. This result is in contrast with numerical results available in literature and casts some doubt on their convergence.

References

[1] Elcrat, Alan; Fornberg, Bengt; Horn, Mark; Miller, Kenneth. Some steady vortex flows past a circular cylinder. *J. Fluid Mech.* **409** (2000), pp. 13–27.

[2] L. Zannetti. Vortex equilibrium in the flow past bluff bodies. *J. Fluid Mech.* to appear, 2006.

ON THE STABILITY OF STRATIFIED QUASIGEOSTROPHIC CURRENTS WITH VERTICAL SHEAR

VALERY N. ZYRYANOV

Water Problems Institute, Russian Academy of Sciences, Russia
E-mail: zyryanov@aqua.laser.ru

The law of conservation of potential vortex in a stratified ocean on the β-plane can be written in the geostrophic approximation in the following form

$$J\Big(P, \Delta P + (P'_z/B^2N^2(z))'_z + by\Big) = 0, \tag{1}$$

where $J(a,b) = a'_x b'_y - a'_y b'_x$ is the Jacobian of two functions, $P(x,y,z)$ is the pressure, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $B = H_0 N/fL$ is the Burger number, $N(z)$ is Brunt- Väisälä frequency, $b = \beta L^2/U_0$, H_0 is the ocean depth, f is the Coriolis parameter, L is the horizontal linear size of the underwater elevation (of the order of a hundred of kilometers), $\beta = df/dy$ is beta-effect, U_0 is the characteristic value of the background flow.

We used the Kozlov-Monin-Neiman-Filyushkin hyperbolic law for $N(z)$ (KMNF-approximation)

$$N(z) = \frac{N_0}{1+\gamma z}. \tag{2}$$

With the effect of wind and the friction at the bed neglected, the boundary conditions will take the form:

$$\text{on the surface of the ocean} \quad z = 0: \quad J(P, P'_z) = 0, \tag{3}$$

$$\text{on the bottom} \quad z = 1: \quad J(P, P'_z - B^2N^2(1)\sigma h(x,y)) = 0, \tag{4}$$

where $\sigma = h_0 f_0 L/H_0 U \approx O(1)$ is a topographic parameter, $h(x,y)$ is a dimensionless perturbation of the bottom. The coordinate axes are directed: x eastward, y northward, z vertically downward. The origin is located on the undisturbed surface of the ocean.

Let us represent pressure $P(x,y,z)$ as the background value $P_\infty(y,z)$, which is determined by the structure of the flow meeting the seamount, and pressure perturbation $\sigma\Psi(x,y,z)$ due to bed topography:

$$P(x,y,z) = P_\infty(y,z) + \sigma\Psi(x,y,z). \tag{5}$$

Let us specify $P_\infty(y,z)$ in the form $P_\infty = -Uy$.

The main equation for pressure perturbation is

$$\Delta\Psi + \left[\frac{\Psi'_z}{B^2N^2(z)}\right]'_z + \lambda(z)\Psi = 0, \tag{6}$$

where function $\lambda(z)$ has the form

$$\lambda(z) = \begin{cases} \dfrac{1}{U(z)}\left\{b - \left[\dfrac{U'_z(z)}{B^2N^2(z)}\right]'_z\right\}, & \text{for} \quad \beta - \text{plane}; \\ -\dfrac{1}{U(z)}\left[\dfrac{U'_z(z)}{B^2N^2(z)}\right]'_z - C_1, & \text{for} \quad f - \text{plane}; \end{cases} \tag{7}$$

and in the general form describes the joint effect of the Earth's sphericity (β-effect), water baroclinicity, and flow velocity shift. The boundary conditions will have the form:

$$\text{at} \quad z = 0: \quad \frac{\partial \Psi}{\partial z} - \frac{U'_z(0)}{U(0)}\Psi = 0; \tag{8}$$

$$\text{at} \quad z = 1: \quad \frac{1}{B^2 N^2(1)} \left[\frac{\partial \Psi}{\partial z} - \frac{U'_z(1)}{U(1)}\Psi \right] + h(x, y) = 0. \tag{9}$$

The stability of some flows was proved. For this purpose we used Arnold's theory.

Theorem 1. Stratified flow with KMNF-approximation of Brunt-Väisälä frequency $N(z)$ will be stable if

$$\min_z \left[\frac{1}{\lambda(z)} \right] > \frac{4B^2 N_0^2}{\gamma^2} \left[\frac{4\pi^2}{\ln^2(1+\gamma)} + 1 \right]^{-1}. \tag{10}$$

Theorem 2. Stratified flow with $N(z) = N_0 = \text{const}$ will be stable if

$$\min_z \left[\frac{1}{\lambda(z)} \right] > \frac{B^2 N_0^2}{\pi^2}. \tag{11}$$

This work was supported by Russian Fund for Basic Researches (Grant 04-05-65058).