HYPER-CHAOS IN PIEZOCERAMIC SYSTEMS WITH LIMITED POWER-SUPPLY

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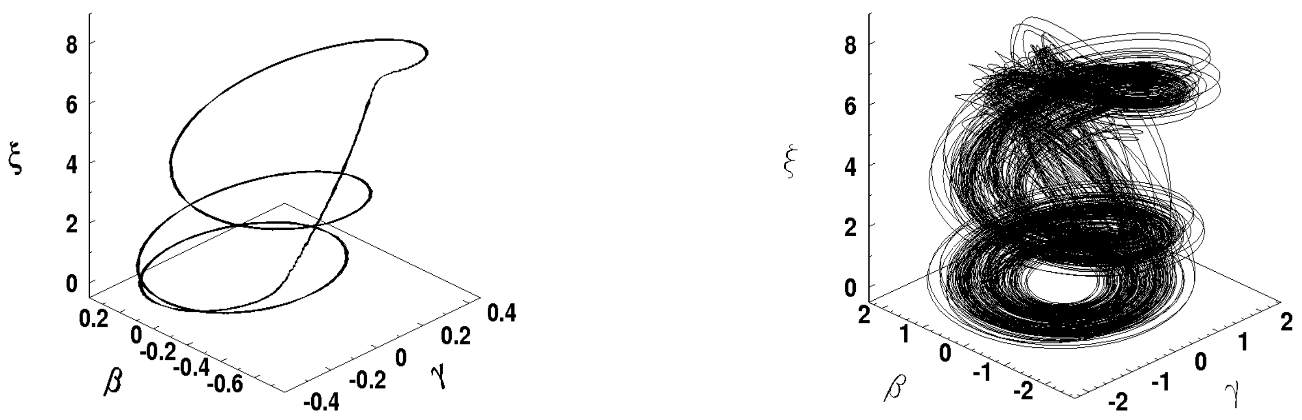
Functioning of many important and mission-critical devices of various engineering machines, including transformers, is based on the effect of coupling of mechanical and electrical fields in piezoceramic mediums. Hence, creation of a general mathematical theory of electroelastic processes in such mediums under arbitrary conditions of mechanical and electrical loading is important, both in scientific and applied aspects. Althogh such theory for many piezoceramic devices and constructions is created (see, for example, Ulitko A. F.'s works [1]) a problem of behaviour of electroelastic fields is considered only for conditions of forced and free oscillations, when the piezoelectric ceramics is under activity of applied mechanical and electrical fields of a priori given values. Thus a problem of influence of dissipation and radiation of energy under oscillations of coupled fields of the device remains outside of many considerations.

The present paper is devoted to the analysis of interaction effects, collectively called the effect of Sommerfeld-Kononenko, in oscillations of piezoceramic transducer and in the mechanism of its excitation - the generator of the electric current of limited power-supply [2, 3]. The new mathematical model of interaction of the generator and the piezoceramic transducer submerged in a hydromedium with resistance is constructed. The coupling of processes in the transformer and the energy source (the generator) leads to such qualitatively new effects in their dynamics as cannot be seen using a model of the problem with unlimited or so-called "ideal" excitation.

In the present work the principal attention was given to examination of origin and development of the deterministic chaos in dynamic systems with limited excitation such as "the piezoceramic transducer - the generator ". The methodic worked up and the large cycle of computer experiments on study of the regular and chaotic regimes interaction systems is carried out. Dependences of spectrums of Lyapunov characteristic exponents on parameters of a system are obtained. The possibility of origin of the deterministic chaos is proved. It is shown, that in a system there are some types of chaotic attractors. Including two types of hyper-chaotic attractors detected.

Phase portraits, sections and Poincaré maps, distribution of invariant measures and spectral densities of attractors of the system are constructed and in details investigated. Some scenarios of transition from the regular regimes to chaotic ones such as, Feigenbaum's cascade and an intermittency of the first type on Pomeau-Manneville are revealed. It is established, that a principal reason of origin of the determined chaos in the system is interaction between subsystems, the transducer and the generator, instead of their individual properties.

In Fig. 1 the regular and hyper-chaotic attractors for the same physics parameters for the cases of an absence and presence of the feed back connection between the subsystems are exhibited.



a b

Fig 1. Projection of the phase portrait of the regular*(a)* and the hyper-chaotic *(b)* attractor.

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THE UNIFIED THEORY OF THE LINEAR SHALLOW WATER EQUATION ON THE ROTATING PLANE

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The linearized system of Shallow Water Equations (LSWE) on a tangent (x, y) plane to the rotating Earth with Coriolis parameter $f(y)$ that depends arbitrarily on the northward coordinate y is considered as a spectral problem of a self-adjoint operator. Despite of its non-constant coefficients the LSWE system can be reduced to a single linear second order equation in x and y. This equation generates easily all the known exact and approximate solutions that arise from different boundary conditions, vanishing of some small terms and specific form of the Coriolis parameter $f(y)$. In particulary in certain limiting cases these solutions reduce to the well-known plane waves of geophysical fluid dynamics: Inertia-gravity (Poincare) waves, Planetary (Rossby) waves and gravity (Kelvin) waves as well as to the steady geostrophic flow. The above approach to the LSWE system applies straightforwardly to the equatorial beta-plane and to the mid-latitude f- and β-planes.

MOTION OF N + 1 VORTICES IN A TWO-LAYER ROTATING FLUID

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Characteristics of motion were studied for $N + 1$ point vortices with A symmetry planes immersed into a two-layer fluid. The central vortex of intensity $\mu\kappa$ is supposed to belong to the upper layer, and an N-gonal configuration of vortices with equal intensity κ - to the bottom one. Theoretically possible stationary movements at $N \geqslant 2$ were analyzed. There is shown:

- as $\mu \geqslant -0.5$, the angular velocity ω and κ have the same sign (the N-gon rotates in the direction which is determined by the intralayer interaction of its vortices), and $|\omega|$ is a monotonously decreasing function of a circumscribed circle radius R (we call such vortex structure an *inverse roundabout*);

- as $\mu < -0.5$, the function ω becomes non-monotonous; it both changes its sign at some value of $R = R_0$ (in this case the rotation of the N-gon in the lower layer is determined by the upper-layer vortex, and the configuration as a whole becomes an *ordinary roundabout*), and takes its minimum value at $R = R_{min}$;

- $R_0(\mu, N)$ and $R_{min}(\mu, N)$ are decreasing functions with respect to μ and increasing ones with respect to N.

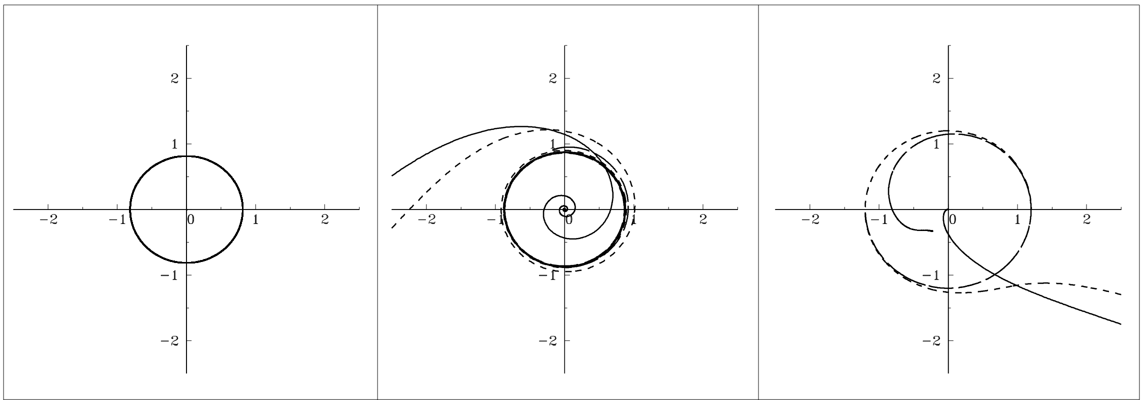
In a particular case $N = 2$, a detailed investigation of possible motion of three vortices was performed; in the initial moment these vortices had more general arrangement (not obviously symmetrical and collinear):

- Use of trilinear coordinates [1] gave the possibility to study qualitatively the relative vortex motion in a large interval of external parameters. There is given a classification of possible motion types, based on the separation of areas with predominant intra- and interlayer interaction of the vortex structure.

- The analysis of stationary states corresponding to singular points of phase portraits was carried out. Dispersion equations were derived for these states, which link the geometrical characteristics of vortex structures with external parameters of the problem.

- There were found new types of stationary solutions: (1) a stable one *eccentric roundabout* - a rigid body rotation (around an immovable vorticity center) of a collinear asymmetric triple vortices, and its particular case - *triton*, having zero total intensity, which moves translationally with a constant velocity; (2) stable and unstable structures in the form of isosceles triangle; (3) stable and unstable symmetric *roundabout* [see, 1 for example, figure where $N = 2$, $\mu = -1$ and $R = 0.815$ - (a), $R = 0.875$ - (b), $R = 1.2$ - (c)].

- We found classes of relative and absolute *choreographies* [2], which correspond to purely periodic relative three-vortex movements.



(a) (b) (c)

Fig 1

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INTERACTION OF INERTIA-GRAVITY WAVES WITH A BAROCLINIC SHEAR FLOW

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The purpose of this paper is to investigate how inertia-gravity waves, *i.e.* internal gravity waves in a rotating medium, may interact with a horizontal flow with both a horizontal and a vertical shear. Such interactions occur everywhere in geophysical flows as soon as inertia-gravity waves propagate in a wind or a current, or encounter a vortical motion such as a large scale vortex. The case of a barotropic shear flow has been addressed by several authors ([5], [3],[1],[6]) and we consider here the case of a baroclinic shear flow.

This problem has been addressed in its full generality within the WKB approximation, without rotation, by [2]. A simple model of wave and baroclinic flow, described in the next section, is introduced in the present paper. We rely on WKB theory and also perform direct numerical simulations of the three-dimensional nonlinear Boussinesq equations. The geophysical motivation of this work is to investigate whether some irreversible wave-induced momentum transport may occur across the shear flow. In the present paper, we explore the wave behavior as it propagates in the shear flow.
Let (x, y, z), with z directed upwards, be a Cartesian coordinate system in the rotating reference frame attached to the fluid container. The baroclinic shear flow consists of a velocity field along the x-direction $\mathbf{U}(y, z)$ in thermal wind balance with a buoyancy field $B(y, z)$. The baroclinic current is a horizontal shear layer, centered about $y = y_s$, with a vertical shear: $U(y,z)/U_0 = \left[1 + \tanh\left(\frac{y - y_s}{L_s}\right)\right]\left[1 + \beta \sin\left(\frac{2\pi z}{H_s}\right)\right] - 1$. The velocity scale is U_0 and the parameter β represents the strength of the baroclinicity: the shear flow is barotropic if $\beta = 0$ or $\beta \ll 1$. This initial condition implies that an inertia-gravity wave packet propagating from a region where $y \ll y_s$ travels, as y increases, from a uniformly translating medium along the x direction with speed $-U_0$, to a moving medium with both a vertical and a horizontal velocity shear. Let $\mathbf{k}$ and $\Omega(\mathbf{k})$ refer to the main wave vector and intrinsic frequency respectively of such a wave packet. We recall that $f \leqslant \Omega(\mathbf{k}) \leqslant N$, where f and N are the Coriolis and local buoyancy frequency. A initial time $t = 0$, we assume that the wave induced energy is confined within a two-dimensional Gaussian envelope along the y and z directions.

We solve the classical ray equations, which describe how a wave vector is refracted by the gradients of the background flow along a ray. From a practical point of view, the ray equations are initialized by a set of rays starting from points that model the wave packet and the wave amplitude is computed from the conservation of wave action.

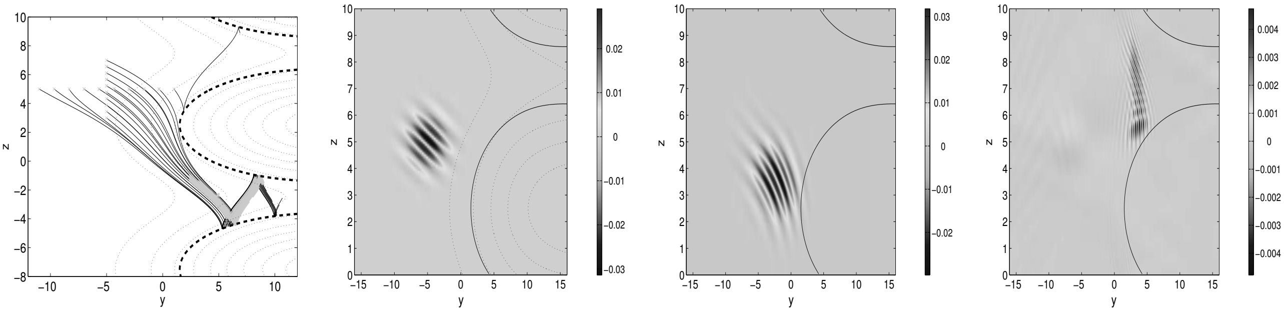
We solve the Navier-Stokes in the Boussinesq approximation in a parallelepipedic domain. The boundary conditions are periodic along the x and z directions and of free slip type along the y direction, so that a pseudo-spectral method can be used. The equations are integrated in time using a third-order Adams-Bashforth scheme.

We have performed several computations, which are described in [4]. The point of view we have chosen is the following: the wave packet propagates into the current such that its intrinsic frequency increases (by Doppler effect) because of the y-dependency of the shear flow. This means that the wave packet should be trapped in the neighbourhood of the $\Omega = N$ surface and possibly break there, at least when $\beta = 0$ [5], [6], thereby inducing mean flow changes. Our purpose in this paper is to investigate the influence of the baroclinicity parameter β on this behavior for $0 < \beta \leqslant 1$.

We illustrate the wave-shear flow interaction for a background flow with a weak horizontal shear, with Rossby number (close to 0.3) comparable to that of the wave packet and for $\beta = 0.5$. The ratio $N_0/f = 4.4$ (where $N_0 = 1$ is the buoyancy frequency of the fluid at rest), the initial Reynolds number of the wave packet and of the background flow are large (of order 10^4) and the Prandtl number is equal to 1.

Ray trajectories predicted from WKB theory are displayed in Fig. 1a. Because the intrinsic frequency Ω increases during propagation, the rays steepen in the neighbourhood of the $\Omega = N$ surface and are trapped there. They next travel downwards with the vertical group velocity. The shear flow we have designed possesses regions where $\partial U/\partial z$ vanishes ($\partial U/\partial y$ being minimum there) so that the $\Omega = N$ surface flattens about this region. This is where the rays, which have become quasi-vertical because Ω is close to N, reflect. Fig. 1a shows that the rays are able to propagate further in the shear flow, within a wave guide limited by two portions of the $\Omega = N$ surface. Results from DNS for the same run are plotted in Fig. 1b to 1d. The same qualitative behavior as in the WKB theory is observed: the wave packet, guided and steepened by the trapping process (Fig. 1c), meets again the $\Omega = N$ surface when it flattens (Fig. 1d). No reflection is observed however: the decay of the wavelength along the y-direction and of the group velocity makes the packet very sensitive to viscous effects. It has totally dissipated by the time the rays first reflect ($t \simeq 300$), assuming the ray predictions remain quantitatively reliable.

In summary, when a wave packet propagates in a baroclinic shear flow such that its intrinsic frequency increases due to the horizontal shear, DNS of the Boussinesq equations show that the packet is trapped



. (a) (b) (c) (d)

Fig 1. (a) Predictions from WKB theory; trajectories of rays starting from points modelling the initial wave packet at $t = 800$. The grey region along the rays indicates where strong amplification occurs. (b) to (d): Results from DNS. Constant contours of the fluctuating density field, at $t = 0$ (b), 56 (c), 176 (d) In all frames, the surface $\Omega = N$ is displayed with a black line.

and dissipates in the vicinity of the $\Omega = N$ surface. When the shear flow locally vanishes (not shown), both WKB theory and DNS results predict that the wave can penetrate into the shear flow through a wave guide. If the wave packet were forced, it could possibly break and thereby induce an irreversible momentum transport across that flow.

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MAGNETIC FIELD GENERATION IN FULLY DEVELOPED TURBULENT FLOW

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We investigate the dynamo problem in the limit of small magnetic Prandtl number (Pm) using a shell model of magnetohydrodynamic turbulence. The model is designed to satisfy conservation laws of total energy, cross helicity and magnetic helicity in the limit of inviscid fluid and null magnetic diffusivity. The forcing is chosen to have a constant injection rate of energy and no injection of kinetic helicity nor cross helicity. We find that the value of the critical magnetic Reynolds number (Rm) saturates in the limit of small Pm. Above the dynamo threshold we study the saturated regime versus Rm and Pm. In the case of equipartition, we find Kolmogorov spectra for both kinetic and magnetic energy except for wave numbers just below the resistive scale. Finally the ratio of both dissipation scales (viscous to resistive) evolves as $\mathrm{Pm}^{-3/4}$ for $\mathrm{Pm} < 1$.

ON AN INTEGRABLE SYSTEM AND SPECTRAL PROPERTIES OF SOME CLASS OF DISCRETE STOURM–LIOUVILLE OPERATORS

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1. It is well-known [1, 2, 3] that there is very close connection between the properties of discrete Stourm–Liouville operators and orthogonal polynomials. E.M.Nikishin [2] was the first who applied the methods from the theory of orthogonal polynomials and Pade approximations to the theory of discrete Stourm–Liouville operator. But in [2] he considered the classical case where $\operatorname{ess\,supp}\sigma = [a, b]$ is a segment on real line $\mathbb{R}$. Here we consider the case when there are some lacunae in $\operatorname{ess\,supp}\sigma$.

2. Let σ be a probability borelean measure, $\operatorname{supp}\sigma \Subset \mathbb{R}$. Consider the Chebyshev expansion the function of type $\widehat{\sigma}(\lambda) := \int \frac{d\sigma(x)}{\lambda - x}$, $\lambda \in \overline{\mathbb{C}} \setminus \operatorname{supp}\sigma$, into continued fraction $\widehat{\sigma}(\lambda) \sim \frac{1}{\lambda - b_1 -}\frac{a_1^2}{\lambda - b_2 -}\frac{a_2^2}{\lambda - b_3 -}\cdots$. Then all the $b_n, a_n \neq 0$ are real. The n-th convergent p_n/q_n to the Chebyshev fraction has the following property $q_n(\lambda)\widehat{\sigma}(\lambda) - p_n(\lambda) = r_n(\lambda)$ where q_n are unequaly determined by the conditions $q_n(\lambda) = k_n\lambda^n + + \cdots$, $k_n > 0$ and $\int q_n(x)q_m(x)\,d\sigma(x) = \delta_{nm}$. The function $r_n(\lambda)$ is the so-called second kind function. Let suppose that $p_{-1}(\lambda) \equiv 1$, $p_0(\lambda) \equiv 0$; $q_{-1}(\lambda) \equiv 0$, $q_0(\lambda) \equiv 1$; $r_{-1}(\lambda) \equiv -1$, $r_0(\lambda) = \widehat{\sigma}(\lambda)$, then the sequences of the three functions will satisfy to the three-term recurrence

$$a_n y_n(\lambda) = (\lambda - b_n)y_{n-1}(\lambda) - a_{n-1}y_{n-2}(\lambda), \qquad n = 1, 2, \dots, \quad a_0 = 1. \tag{1}$$

3. Let $\{e_j\}$ be standart basis in $\ell^2(\mathbb{N})$. Define [2] a *discrete Stourm–Liouville operator* $J : \ell^2(\mathbb{N}) \to \ell^2(\mathbb{N})$ by the equalities

$$\begin{aligned} Je_1 &= a_1e_2 + b_1e_1, \\ Je_n &= a_ne_{n+1} + b_ne_n + a_{n-1}e_{n-1}, \qquad n = 2, 3, \dots; \end{aligned} \tag{2}$$

then σ is the spectral measure and $\operatorname{supp}\sigma$ is the spectrum of J.

When $\operatorname{ess\,supp}\sigma = [-1, 1]$ the sequance $\{w_n(z)\}_{n\in\mathbb{N}_0}$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $w_0(z) \equiv 1$, $w_n(z) = \langle e_n, (\lambda - - J)^{-1}e_1\rangle$, $n = 1, 2, \dots$, $|z| < 1$, $\lambda = (z + 1/z)/2 \notin \operatorname{supp}\sigma$, is called [4] the *Weyl solution* of 1. By the spectral theorem we obtain $w_n(z) = r_{n-1}(\lambda)$, $\lambda \notin \operatorname{supp}\sigma$.

4. Now we shall set $S := \bigsqcup_{j=1}^{g} \Delta_j$, $\Delta_j = [e_{2j-1}, e_{2j}]$, $j = 1, 2, \dots, g+1$, polynomial $h(\lambda) = \prod_{j=1}^{2g+2}(\lambda - - e_j)$. We suppose that measure σ has the following form $d\sigma(x) = \rho(x)dx/\sqrt{-h(x + i0)} + \sum_{k=1}^{m} \rho_k\delta(x - - a_k)$, where $\rho \neq 0$ is holomorphic on S, the square root follows the condition $\sqrt{h(z)}/z^{g+1} \to 1$, $z \to \infty$, $z \in D := \overline{\mathbb{C}} \setminus S$, all $\rho_k > 0$, $a_k \in \mathbb{R} \setminus \widehat{S}$, where $\widehat{S}$ is the convex hull of S.

Our main result is the following

Theorem 1 *Under the above condition on σ the following formulae holds*

$$q_n(\lambda)r_n(\lambda) = \frac{\prod_{j=1}^{g}(\lambda - \lambda_j(n))}{\sqrt{h(\lambda)}} + o(\delta^n), \quad \lambda \in \widehat{\mathbb{C}} \setminus \operatorname{supp}\sigma, n \to \infty, \delta \in (0, 1),$$

where values $\lambda_1(n), \dots, \lambda_g(n)$ come from the solutiion $\lambda_1(t), \dots, \lambda_g(t)$ of an integrable system of g equations when $t = n \in \mathbb{N}$; all values $\lambda_j(t) \in [e_{2j}, e_{2j+1}]$ when $t \geq 0$.

The proof of the above thorem is bases on the special, see [5, 6], method to investigate asymptotic properties of the polynomial orthogonal on several segments.

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INERTIAL PULSATIONS OF LENS-LIKE STRATIFIED ANTICYCLONIC VORTICES

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Hydrostatic, stratified Boussinesq primitive equations (PE) are widely used for modeling large- and mesoscale variability in planetary atmospheres and oceans. However, exact nonstationary solutions for the PE are limited. Some analytic solutions were found for finite-area lens-like vortices in the reduced-gravity shallow water formulation (see Rubino *et al.* [1] and references therein). One family of so-called rodon and pulson solutions is described by a set of ordinary differential equations when velocities are assumed to be linear functions of the horizontal coordinates, so that both horizontal divergence and vorticity are spatially uniform within the vortex area. This class of exact solutions describes rotation and pulsations of elliptical anticyclonic eddies with maximum velocity at the vortex boundary.

Circular vortices with more realistic horizontal and vertical structure are also able to support nonlinear pulsations with inertial frequency as described analytically for the shallow water model by Rubino *et al.* [1]. In this second family of analytical nonstationary solutions the divergence of velocity oscillates in time being spatially uniform within the vortex boundaries while the vorticity may depend on time and coordinates. The second family of reduced-gravity analytical pulson solutions has a self-similar nature and can be extended to rather arbitrary radial and vertical vortex structure in continuously stratified PE [3]. In the self-similar form the pulson solution describes radial expansion and contraction of the vortex which maintains the same spatial structure in Lagrangian coordinates.

We consider an axisymmetric stratified flow with horizontal velocity $\mathbf{v} = (u, v)$ in cylindrical coordinates (r, θ, η), where $\eta = (\rho - \rho_0)/\rho_0$, assuming density, ρ, increases monotonically downward from a reference value ρ_0. The hydrostatic balance in such isopycnal coordinate system can be written as $\partial\phi/\partial\eta = gz$, $\phi \equiv p/\rho_0 + gz\eta$. Here ϕ is the Montgomery potential, p is the pressure, g is the gravity acceleration, and $z(t, r, \eta)$ represents the depth of isopycnal surfaces.

Inviscid flows with density conserved by individual fluid parcels conserve also the absolute angular momentum $m \equiv vr + fr^2/2$ (f is the Coriolis parameter). We assume that the flow is located inside the area which may depend on time, t, and seek the solution for the angular momentum in the form $m = M(R, \eta)$, $R = r/\sqrt{S}$, where $S(t)$ is the area expansion coefficient. In this case, radial velocity depends linearly on the radial distance, r

$$u = -\frac{\partial m}{\partial t}\left(\frac{\partial m}{\partial r}\right)^{-1} = \frac{\dot{S}r}{2S}, \tag{1}$$

so that it satisfies the condition of zero horizontal velocity at the vortex center $r = 0$. Note, that this radial velocity does not depend on η and the horizontal divergence is spatially uniform: $\nabla \cdot \mathbf{v} = \dot{S}/S$. Therefore, such vortex dynamics is described by the shallow water model for each η

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi u}{\partial r} + \frac{\phi u}{r} = 0, \tag{2}$$

where the solutions for ϕ and z have the same form

$$\phi = \frac{1}{S}\Phi(R,\eta), \qquad z = \frac{1}{Sg}\frac{\partial \Phi}{\partial \eta}. \tag{3}$$

The relation between M and Φ is obtained from the radial momentum balance

$$\frac{M^2}{R^4} - \frac{1}{R}\frac{\partial \Phi}{\partial R} = \frac{1}{4}(f^2S^2 - \dot{S}^2 + 2S\ddot{S}) \equiv \frac{f^2}{4}(1-a^2), \tag{4}$$

where the RHS remains constant if $S = 1 + a\sin(ft)$, where $0 < a < 1$ in order to satisfy the physically realistic demand that $S > 0$ (for $a = 0$ Eq. (4) described gradient wind balance in a stationary vortex). Inertial oscillations in this set of nonlinear nonstationary solutions depend on a, while the spatial distribution $\phi(R,\eta)$ in coordinates (R,η) is the same as for the stationary solution except its amplitude pulsates inversely proportional to S according to Eq. (3) in order to provide the mass conservation described by Eq. (2). Therefore, it has physical meaning only for finite area vortices $\Phi = 0$ for $R > R_0$; the actual vortex radius pulsates with time as $r_0 = R_0\sqrt{1 + a\sin(ft)}$. Correspondingly, the isopycnal surfaces become deeper or shallower following pulsations in r_0. Such solution can describe anticyclonic (warm-core) lens-like vortex with all isopycnals outcroping at the same level $z(R_0,\eta) = 0$ at variable radial distance r_0.

Note that the vertical velosity $w = -\dot{S}z/S$ increases from zero at the reference level to maximum at the lowest isopycnal while Φ decreases from maximum at the reference level to zero at the lowest isopycnal overlying deep motionless fluid as often assumed in the reduced-gravity approximation. Correspondingly, azimuthal velocity calculated from (4) for $a > 0$ deviates from stationary gradient balance to compensate impact of pulsating radial velocity. Thus, such unbalanced solution has nonzero agradient velocity (cf. Sutyrin [2]) and remains unbalanced because inertia-gravity waves are trapped inside the edge of such lens-like vortex: they are not able to propagate through outcroping isopycnals.

These self-similar solutions demonstrate that during the inertial period the structure of axisymmetric pulsons remains essentially the same in properly normalized isopycnal coordinates. The simple analytic expression for nonstationary PE solutions is found for fairly arbitrary horizontal and vertical vortex structure starting from a stationary lens-like anticyclone and depending on the amplitude, a, of the vortex area pulsations. These exact solutions can be used for assessing laboratory and numerical models with layer outcroping.

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EVOLUTION OF AN INTENSE BAROCLINIC VORTEX IN A SHEARED FLOW

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Intense baroclinic vortices are abundant in the ocean and their evolution without any background current has been the subject of many investigations (see the recent review by Carton [1], and references therein). On the spherical earth for instance, it is well known that oceanic vortices are strongly influenced by a barotropic potential vorticity (PV) gradient called planetary β-effect. The effects of a background current on the propagation of vortices depend on the current structure. Under the influence of a constant barotropic current, vortices are merely advected at the current velocity. The advection speed associated with a vertically sheared background current has been considered by Hogg and Stommel [2] or Marshall and Parthasarathy [4]. Besides this direct advective effect on the vortex, a vertically sheared current is generally associated with a *baroclinic* PV gradient that results in the modification of the β-gyres and induces an additional vortex displacement. Kaz'min and Sutyrin [3] and Vandermeirsch et al. [9] have shown that this effect, refered to as a "*baroclinic* β-effect" because of its similarity with the planetary β-effect, is important and could drastically modify the vortex trajectory.

Large-scale oceanic currents have both vertical and horizontal shear. Uniform horizontal shear is known to produce elliptical deformation of the vortex core (e.g., Sutyrin et al. [8]) while non-uniform shear may result in higher azimuthal mode deformation (Sutyrin and Carton [5]). The present study focuses on the net influence of a horizontally and vertically sheared flow, typical for a large-scale Rossby wave, on the evolution of propagation of intense vortices, taking both advection and baroclinic β-effect into account. By simplifying the flow and vortex structure in a one-and-a-half layer, the present study is only a first step in elucidating the influence of sheared flows on coherent vortices in the ocean.

We develop an analytical theory for the motion of an intense vortex in the presence of a sheared flow on the f-plane and the beta-plane. We derive asymptotic expansions and compare them with numerical simulations in the framework of the reduced-gravity quasi-geostrophic model. The analytical method suggested for intense vortices with piecewise-constant potential vorticity (Sutyrin and Flierl [6]; Sutyrin and Morel [7]) is generalized to take into account a large-scale sheared flow with nonuniform background potential vorticity.

The theory describes the vortex advection by the flow and the vortex drift due to the background potential vorticity gradient. The net advective effect of the flow on the vortex is found to be much less than the maximum of the flow velocity due to the baroclinic beta-effect and the horizontal shear. Besides known elliptical core deformations, triangular deformations are generated described by the third azimuthal mode at the core boundary. Additionally on the beta-plane, the planetary beta-effect provides predominantly westward vortex drift.

The asymptotic theory is shown to agree well the results of a numerical pseudo-spectral, high-resolution biperiodic model when the vortex velocity is much larger than the maximm velocity of the flow represented by a zonal Rossby wave. Both meridional and zonal vortex drifts are slightly overestimated when the flow velocity is comparable with the vortex velocity. Vortex size is shown to be more influential on vortex trajectory than the Rossby wave length. In particular, smaller vortices drift westward farther and faster than large ones. Vortex core deformations contain the typical modes 2 and 3 in general; stronger mode 3 component for less intense vortex as predicted by the theory.

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NEW METHOD OF CONSTRUCTION OF SOLUTIONS OF THE QUASILINEAR PARABOLIC EQUATIONS IN THE PARAMETRICAL FORM

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The algorithm is formulated in the conditional form in the supposition, that all functions exist and have those a smoothness, which is required that the algorithm could be applied. Let's consider the quasilinear parabolic equation with a small parameter ε

Let's assume, that connection of solutions (it is non standard change of variables) of the equation (1) with equation (9)

It is possible to select as

$$\varepsilon Z_t - \varepsilon^2 (K(Z,\varepsilon) Z_x)_x + F(Z,\varepsilon) = 0, \tag{1}$$

$$Z(x,t,\varepsilon)|_{x=x(\xi,\delta),t=t(\xi,\delta)} = U(\xi,\delta,\varepsilon) \tag{2}$$

for want of it the function $Z(x,t,\varepsilon)$ is a solution (1). Let's assume that the determinant (Jacobian) $detJ$ of this change of variables is different from zero in some area in R. Let's assume, at least locally, there is a reconversion $\xi = \xi(x,t,\varepsilon), \quad \delta = \delta(x,t,\varepsilon)$. Examples show, that there are cases when these suppositions are correct.

Let's assume, that the ratio for streams

$$K(Z(x,t,\varepsilon),\varepsilon)\frac{\partial Z}{\partial x}|_{x=x(\xi,\delta,\varepsilon),t=t(\xi,\delta,\varepsilon)} = Y(\xi,\delta,\varepsilon), \quad K(Z(x,t,\varepsilon),\varepsilon)\frac{\partial Z}{\partial t}|_{x=x(\xi,\delta,\varepsilon),t=t(\xi,\delta,\varepsilon)} = T(\xi,\delta,\varepsilon) \tag{3}$$

We are calculating derivatives in right members (3) we shall receive of equalities

$$\frac{K(U(\xi,\delta,\varepsilon),\varepsilon)}{detJ}\left(\frac{\partial U}{\partial \xi}\frac{\partial t}{\partial \delta} - \frac{\partial U}{\partial \delta}\frac{\partial t}{\partial \xi}\right) = Y(\xi,\delta,\varepsilon), \quad \frac{K(U(\xi,\delta,\varepsilon),\varepsilon)}{detJ}\left(-\frac{\partial U}{\partial \xi}\frac{\partial x}{\partial \delta} + \frac{\partial U}{\partial \delta}\frac{\partial x}{\partial \xi}\right) = T(\xi,\delta,\varepsilon) \tag{4}$$

Let's multiply the equation (1) on $K(Z,\varepsilon)$. After substitutions (3) in the equation (1) with allowance for (2) we shall receive the equation

$$\varepsilon T(\xi,\delta,\varepsilon) - \frac{\varepsilon^2 K(U,\varepsilon)}{detJ}\left(\frac{\partial Y}{\partial \xi}\frac{\partial t}{\partial \delta} - \frac{\partial Y}{\partial \delta}\frac{\partial t}{\partial \xi}\right) + K(U,\varepsilon)F(U,\varepsilon) = 0. \tag{5}$$

By virtue of a smoothness of function$Z(x,t,\varepsilon)$. The equality $Z_{xt}=Z_{tx}$ can be noted as

$$-\frac{\partial}{\partial\xi}\left[\frac{Y}{K(U,\varepsilon)}\right]\frac{\partial x}{\partial\delta}+\frac{\partial}{\partial\delta}\left[\frac{Y}{K(U,\varepsilon)}\right]\frac{\partial x}{\partial\xi}-\frac{\partial}{\partial\xi}\left[\frac{T}{K(U,\varepsilon)}\right]\frac{\partial t}{\partial\delta}+\frac{\partial}{\partial\delta}\left[\frac{T}{K(U,\varepsilon)}\right]\frac{\partial t}{\partial\xi}=0. \quad (6)$$

Theorem 2 The system of the nonlinear algebraic equations (3)- (6) rather variables has a unique solution:

$$\frac{\partial x}{\partial\xi}=\frac{K}{YP(\delta,\xi,\varepsilon)}\left[-Q_1Q_2TU_\xi+Q_1FKTU_\xi^2+Q_1T^2U_\xi^2+Q_2^2TU_\delta-Q_2FKTU_\delta U_\xi-Q_2T^2U_\delta U_\xi-\right.$$
$$\left.-Q_1T_\xi U_\xi Y^2+Q_2T_\delta U_\xi Y^2\right], \quad (7)$$

$$\frac{\partial x}{\partial\delta}=\frac{\partial K}{YP(\delta,\xi,\varepsilon)}\left[-Q_1^2TU_\xi+Q_1Q_2TU_\delta+Q_1FKTU_\delta U_\xi+Q_1T^2U_\delta U_\xi-Q_2FKTU_\delta^2-\right.$$
$$\left.-Q_2T^2U_\delta^2-Q_1T_\xi Y^2U_\delta+Q_2T_\delta Y^2U_\delta\right],$$
$$\frac{\partial t}{\partial\xi}=\frac{Q_2K[Q_1U_\xi-Q_2U_\delta]U_\xi}{P(\delta,\xi,\varepsilon)},\quad \frac{\partial t}{\partial\delta}=\frac{Q_1K[Q_1U_\xi-Q_2U_\delta]}{P(\delta,\xi,\varepsilon)},$$
$$P(\delta,\xi,\varepsilon)=Q_1FKTU_\xi+Q_1T^2U_\xi-Q_2FKTU_\delta-Q_2T^2U_\delta-Q_1T_\xi Y^2+Q_2T_\delta Y^2, \quad (8)$$

where

$$Q_1=F(U)K(U)U_\delta+\varepsilon TU_\delta-\varepsilon^2YY_\delta,\quad Q_2=F(U)K(U)U_\varepsilon+\varepsilon TU_\xi-\varepsilon^2YY_\xi.$$

Theorem 3 The necessary condition of a resolvability of the redefined system (7), (8) is equality of the mixed derivatives $x_{\xi\delta}=x_{\delta\xi}, t_{\xi\delta}=t_{\delta\xi}$. These **two conditions coincide** for want of any smooth functions $U(\xi,\delta,\varepsilon)$, $T(\xi,\delta,\varepsilon)$, $F(U,\varepsilon)$, $K(U,\varepsilon)$ and are by partial equation of the second order concerning function $U(\xi,\delta)$

$$LU=\frac{\partial}{\partial\delta}\left(\frac{Q_2K(U,\varepsilon)(Q_1U_\xi-Q_2U_\delta)U_\xi}{P(\delta,\xi,\varepsilon)}\right)-\frac{\partial}{\partial\xi}\left(\frac{Q_1K(U,\varepsilon)(Q_1U_\xi-Q_2U_\delta)U_\xi}{P(\delta,\xi,\varepsilon)}\right)=0. \quad (9)$$

Thus, there is a new method of construction of solutions of the equation (1) in the parametrical form with two parameters on the following algorithm.

a) We set a concrete form of functions $F(U,\varepsilon)$, $K(U,\varepsilon)$, T, Y in the equation (1).

b) We calculate coefficients of the equation (9).

c) We discover one from solutions homogeneous equation (9) with distinct from zero by a Jacobian.

d) We calculate derivatives under the formulas (7) - (8).

Further, we calculate functions $x=x(\xi,\delta)$, $t=t(\xi,\delta)$. The method is distributed to the quasilinear hyperbolic equations.