RUBBER ROLLING: GEOMETRY AND DYNAMICS OF 2-3-5 DISTRIBUTIONS

JAIR KOILLER[1], KURT EHLERS[2]

[1]Fundação Getulio Vargas, Brazil

[2]Truckee Meadows Community College, USA

E-mail: jkoiller@fgv.br

Nowadays, the mathematical structure of holonomic mechanical systems is understood profoundly, thanks to the mathematics genealogy going back to Euler, Lagrange, Jacobi, Hamilton and Liouville. All this work lead to the beautiful geometrical structures of symplectic and Poisson geometries.

In contradistinction, the mathematical structure of nonholonomic systems is still a "developing country". We address here two lines of research.

The first concerns the affine connection approach advocated by E. Cartan in a communication at the 1928 International Congress of Mathematicians, where he introduced the notion of a nonholonomic affine connection $\nabla_{\mathcal{D}}$. He showed that the equations of motion for a mechanical system with nonholonomic constraints can be expressed using an affine connection expressed in terms of a coframe over the n-dimensional configuration manifold adapted to the kinetic energy and constraint distribution. For a particular mechanical system there is a family of adapted coframes all leading to the same equations of motion. The family is parameterized by a Lie subgroup of $Gl(n)$. A fundamental question is to determine differential invariants uniquely associated to a family of coframes. A second fundamental problem is to deterimine the (local) symmetry algebra of the system. The maximally symmetric systems are those for which the differential invariants are constant. In this case the differential invariants are the structure constants for the Lie algebra of symmetries. Cartan essentially settled the strongly nonholonomic case, where $\mathcal{D} + [\mathcal{D}, \mathcal{D}] = \mathcal{TQ}$ using his equivalence method, but discouraged the study of nonholonomic connections whose distributions are not strongly nonholonomic, predicting that it would be difficult to construct the invariants.

Nonetheless, our previous work on Engel (2-3-4) distributions, which became tractable because of refinements in Cartan's methodas well as computer algebra systems, motivated us to pursue the next case in line, 2-3-5 distributions, which have some remarkable aspects. In his famous five variables paper Cartan applied his method of equivalence to make an extensive study of nonintegrable rank to distributions defined on five-dimensional manifolds. Among his results, Cartan showed that maximally symmetric rank two distributions have a symmetry algebra isomorphic to the 14 dimensional exceptional Lie algebra of non-compact type $\mathcal{G}_{\in}$. This distribution is physically realized by the rolling of a sphere of radius a over a sphere of radius $3a$. There has been renewed interest in this paper in recent years. Montgomery and Bor give an explicit description of the infinitesimal action of $\mathcal{G}_{\in}$ on the ball-ball system and Zelenko gave a geometric interpretation of the fundamental differential invariant constructed by Cartan.

In the first part we apply Cartan's method of equivalence to determine the fundamental invariants of a mechanical system define on a 5-dimensional configuration manifold with a rank two nonholonomic constraint. We also determine a bound on the dimension of the symmetry algebra of such a system. In the second part we look at the nonholonomic dynamics of a convex body rolling without sliping or twisting on a surface. The associated distribution has growth 2-3-5. The Lagrange-d'Alembert equations describe the motion of "rubber" coated bodies, in contradistinction with "marble" bodies that have an extensive literature. We call attention to the situation where the "rubber" body rolls over a sphere. It is a generalized Chaplygin system and the dynamics reduces to $T * S^2$ with a non-closed 2-form.

Our primary example is that of a (rubber) Chaplygin ball, balanced but dynamically asymmetric: the principal moments of inertia may be unequal. In the sphere-sphere case, it is actually a $L - R$ Chaplygin system, and the 2-form is conformally symplectic: the reduced system is Hamiltonian after a coordinate dependent change of time scale. In particular, there is a smooth invariant volume.

We present some conjectures about possible integrable cases, and we inquire if the 3:1 ratio keeps some vestige of the $\mathcal{G}_\in$ distributional symmetry.

HOMOCLINIC AND PERIODIC ORBITS IN HAMILTONIAN SYSTEMS WITH A SADDLE-CENTER EQUILIBRIUM

OKSANA Y. KOLTSOVA

Deptartment of Comput. Math. and Cybernetics, Nizhny Novgorod State University, Russia
E-mail: koltsova@uic.nnov.ru

Consider a family of real analytic two degrees of freedom Hamiltonian systems with a saddle-center equilibrium p, $H_\mu(p) = 0$, $\mu \in R^2$. Suppose for $\mu = 0$ the corresponding Hamiltonian system has a homoclinic loop Γ to p. Evidently, for $\mu \neq 0$ the loop is already destroyed, but some multi-round loops can exist. The present result concerns the existence of homoclinic orbits of the roundness $2^k \cdot 3^m$ ($k, m \in Z^+$, $k + m \geqslant 1$) in nonresonance case.

We also prove that in the nonresonance case in some neighbourhood of every such multi-round homoclinic orbit there exist four countable families of one-round periodic orbits, accumulating at the homoclinic orbit.

This work was supported by CRDF (grant RU-M1-2583-MO-04).

ESTIMATION OF OPTIMAL FOR CHAOTIC TRANSPORT FREQUENCY OF NONSTATIONARY FLOW OSCILLATION

KONSTANTIN V. KOSHEL[1], DMITRY V. STEPANOV[1], YURY G. IZRAILSKY[2]

[1]Pacific Oceanological Institute, Far East Branch of
Russian Academy of Sciences, Russia
[2]Institute of Automation and Control Processes, Far East Branch of
Russian Academy of Sciences, Russia
E-mail: kvkoshel@poi.dvo.ru

Three models have been investigated: two barotropic models, where submerged obstacles had axisymmetric and elliptic forms respectively, and one two-layer model with the delta-shaped obstacle in the bottom layer, which produces a point topographic vortex. When we set periodic oscillations of the running current for the above cases, we obtain three different dynamic systems, each one having 3/2 degrees of freedom. It's well known, that chaotic properties of such dynamical systems strongly depend on frequency of flow oscillations. We carried out direct numerical simulation of chaotization in the above systems for a wide range of oscillation disturbance frequencies and found that for chaotization degree there exist besides the global maximum several local maxima. In the present work, we performed new highly accurate calculations for barotropic models with Gauss and elliptic obstacles examined earlier. This new study has shown also local maxima, but very weak. The mechanism of influence of flow oscillation frequency on chaotization degree was explained in terms of nonlinear resonances. It was shown, that the optimum frequency for chaotization is equal to the maximum frequency of the fluid particle rotation in the vortex; i.e. this is the frequency where the nonlinear resonance, multiple to 1, vanishes in the corresponding system. Other local maxima of chaotization correspond to disturbance frequencies, where nonlinear resonances with multiplication more or less than 1, vanish. It was shown that the optimum frequency range for trajectory chaotization lies between the maximum frequency of the rotation of the fluid particle and a half of this maximum frequency.

KHOKHLOV-ZABOLOTSKAYA-KUZNETSOV TYPE EQUATION: IN HETEROGENEOUS MEDIA

ILYA KOSTIN, GRIGORI PANASENKO

Laboratory of Mathematics of the University of Saint-Etienne (LaMUSE), France
E-mail: grigory.panasenko@univ-st-etienne.fr

The so called Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation belongs to the set of non-linear acoustics models, such as the well-known Riemann wave equation (or the non-linear transfer equation), Burgers equation, Korteweg–de Vries (KdV) equation, Khokhlov–Zabolotskaya (KZ) equation, Zakharov–Kuznetsov equation and Rudenko–Sukhorukov equation (see about these equations, for example, the book B. K. Novikov, O. V. Rudenko, V. I. Timoshenko *Nonlinear Underwater Acoustics*, Amer.Inst.of Physics, New-York, 1987). These models are derived from the linear or non-linear wave equation for the acoustic pressure, usually, under the hypothesis of small variations of this pressure. More precisely the KZK equation has the form

$$\alpha u_{z\tau} = (f(u_\tau))_\tau + \beta u_{\tau\tau\tau} + \gamma u_\tau + \Delta_x u, \tag{1}$$

where $u_\tau = u_\tau(z,x,\tau)$ is the acoustic pressure, $(z,x) \in I\!R^\times I\!R^d$, $d = 1,2$ are space variables and τ is the retarded time. The nonlinear function f in the KZK equation is quadratic, i.e., $f(s) = = \theta s^2$, although for the description of space-limited beams subject of the diffraction and self-action effects it can be taken as cubic: $f(s) = \theta s^3$ (see the same book) and so, in the real physical setting, f may have a more complicated shape. On the other hand, all these models are derived under the assumption of small oscillations of the pressure, and so, one can always consider f as quadratic or cubic for $|s| \le s^*$ and anything different for $|s| > s^*$ for some finite s^*. If $|u_\tau|$ is smaller than s^*, then the two models (with $f(s) = \theta s^2$, $\forall s$ and $f(s) = \theta s^2$ for $|s| \le s^*$) coinside. The advantage of this modified shape of f is that (as it will be proved below) we can get the global existence theorem as soon as f has bounded derivative.

These arguments motivate us to consider the "KZK type equation", that is, equation (1) with a nonlinearity f admitting a bounded derivative. Let us emphasize that this shape gives a more convenient physical description than the classical quadratic shape.

Another particularity of the model we consider in the present paper is that the coefficients are rapidly oscillating functions of z. This corresponds to the heterogeneous (stratified in the direction of the axis z) acoustic media. This feature complicates the problem, although it allows us to apply the homogenization method (see for example, the book N. S. Bakhvalov, G. P. Panasenko *Homogenisation: Averaging Processes in Periodic Media*), Nauka, Moscow, 1984 and Kluwer, Dordrecht—Boston–London, 1989) to obtain the homogenized model. Its solution is close to the one of the initial problem.

It seems that so far there was no any publication on the existence and uniqueness of the solution for the KZK (or KZK type) equation, although the authors discovered that independently and simultaneously these questions (as well as the derivation of the KZK equation form the Navier-Stokes model) in the case of constant coefficients were studied by C. Bardos and A. Rozanova. They consider the KZK equation in the whole space ($x \in I\!R^d$) in the case of constant coefficients. In the present study we shall consider the varying (and even rapidly oscillating) coefficients of the KZK type equation set for $x \in \omega$, where $\omega \subset I\!R^d$ is a bounded domain. The boundedness of f' will ensure the global existence. In the case of stratified media we homogenize the KZK type equation and prove the closeness of the solutions of the homogenized and initial models.

These results can be applied in underwater or atmosphere acoustics.

EVALUATION OF PETROLEUM PATCH TRANSPORT IN COSTAL ZONES BY MATRIX METHODS

TATYANA S. KRASNOPOLSKAYA[1], VYATCHESLAV V. MELESHKO[2]

[1]Institute of a hydromechanics of NASU, Ukraine
[2]Kiev National Taras Shevchenko University, Ukraine
E-mail: kras@ihm.kiev.ua

The study of transport of pollutants is an important issue. This work deals with exchange matrix method described by Spencer and Wiley [3]. We apply this method to problems of petroleum patch transport in tidal flows. A good approximation of such flows was suggested by Zimmerman [4]. He deviated from earlier attempts at modeling tidal flows by means of turbulence theory and adopted the idea of chaotic advection, first put forward by Aref [1]. Zimmerman suggested a kinematical model which is based upon a superposition of a tidal and a residual current flow field. The kinematical model describes a motion of a mathematical point that moves at each instant with the velocity corresponding to its instant position. The petroleum particle is supposed to be inertialess, not subjected to diffusion or interfacial tension. The residual time independent current field is an infinite sequence of clockwise and anticlockwise rotating eddies. Streamlines of the residual velocity field divide the whole area into square cells of equal area, with elliptic points in the centers and hyperbolic points in the corners of cells. The tidal field plays a role of a perturbation of the Hamiltonian system (the stream function is the Hamiltonian for the residual flow). In general this perturbation leads to chaotic dynamics and could be studied in different ways, for example, by means of Poincar'e sections. However, in pollutant spreading problems, we are interested in the short term history of pollutant transport. Poincaré sections present the history of motion of points in some area during a long time interval, say, during a thousand periods of a tidal flow. (For the problem of petroleum patch transport this corresponds to the history of one point during almost one and a half year.) On the contrary, ecological considerations demand that disastrous spread of pollution has to be stopped in days or weeks. Therefore, we need to know which part of the Eulerian space will be polluted in a short time and, more importantly, how much petroleum will leak to some specific part of a sea. The orbit expansion method, developed for a quantification of the chaotic transport and exploited an assumption that the contributions of tidal and residual currents are of different orders (the tidal is much stronger), does not give answers to those questions. In our case, it is important to know not the mixing region (where presumably mixing is instantaneous) obtained by the long time tool Poincaré sections, – or the rate of material exchange (which could be high in a very narrow domain), but how uniformly this mixing region is distributed over the whole area during a specific finite interval of time.

We suggest a different approach for an estimation and quantification of pollutant transport, based on the statistical quantities such as a coarsegrained density [2]. First step is to present a petroleum patch, for example, as a circular blob continuously occupying some part in marine or coastal zone. Then we use a contour tracking algorithm [2] to find the blob's boundary in Eulerian space at any moment of time. Knowing the position of the contour line (the boundary of the petroleum patch) we can construct an Eulerian description of the mixing process, giving an opportunity to quantify mixing at any moment of time. The second step is to divide the whole costal area of investigating by cell grid in the square boxes with the side size δ and the area $S_\delta = \delta^2$. We may number all cells starting from 1 to N . The next step is to compute the exchange matrix coeffcients D_{ij} using Spencer and Wiley method.

Coefficient D_{ij} is equal to the fraction of the petroleum originally occupying completely the j box which is moving by flow field to the i cell. This is the basic step in the matrix method. In order to compute the value of coefficient D_{ij} we put petroleum patch as a square blob continuously occupying the j cell. Then we use a contour tracking algorithm that conserves both area and topological properties (connectedness and non selfintersection) to find the blob's boundary in

Eulerian space under investigation at the end tidal cycle. Then we project the found blob's boundary in to the i cell. The ration of dyed material in that i cell $S_b^{(i)end}$ to the area of initial dyed blob $S_b^{(j)start}$ in the j box is equal D_{ij} , namely

$$D_{ij} = \frac{S_b^{(i)end}}{S_b^{(j)start}} = \frac{S_b^{(i)end}}{S_\delta} \tag{1}$$

Then using this matrix we can predict transport of petroleum from any place (any box) in the area to an arbitrary location and determine the time when it happens. If $a_j^{(0)}$ is the initial course grained density in the j cell, the density in the i cell after n cycles are given by the elements of the matrix

$$[a_i^{(n)}] = [a_j^{(0)}][D_{ij}]^n = [a_j^{(0)}][D_{ij}^{(n)}] \tag{2}$$

If $a_i^{(n)}$ is not zero then petroleum polluted the i cell and n number shows after how many tidal cycles it is happened. For zero $a_i^{(n)}$ it is necessary to have either zero value of the $a_j^{(0)}$ or $D_{ij}^{(n)}$. We want to stress that for computing of $D_{ij}^{(n)}$ we need to know flow field. It could be done by analytical presentation or by numerical approximation or even by experimental observations.

References

[1] Aref, H. Stirring by chaotic advection. *J. Fluid Mech.* **143** (1984), pp. 1–24.

[2] Krasnopolskaya, T. S.; Meleshko, V. V.; Peters, G. W. M.; Meijer, H. E. H. Mixing in Stokes flow in an annular wedge cavity. *Eur. J. Mech.B/Fluids* **18** (1999), pp. 793–822.

[3] Spencer, R. S.; Wiley, R. M. The mixing of very viscous liquids. *J. Colloid Sci.* **6** (1951), pp. 133–145.

[4] Zimmerman, J. T. F. The tidal whirlpool: a review of horizontal dispersion by tidal and residual currents. *Neth. J. Sea Res.* **20** (1986), pp. 133154.

RESOLUTION OF NEAR-WALL PRESSURE IN TURBULENCE ON THE BASIS OF FUNCTIONAL APPROACH

EFIM B. KUDASHEV

Space Research Institute, Russian Academy of Sciences, Russia

E-mail: kudashev@iki.rssi.ru

Efficient experimental technique and tools for study of the physical processes in view are developed, as well as experimental equipment and new methods for measurements of turbulent fluctuations in the turbulent boundary layer. The fluctuating wall pressures that develop beneath a turbulent boundary layer have received extensive analytical and experimental investigation over the past few decades [1, 2]. In fluid mechanics and hydrodynamical acoustics the principal interest in turbulent pressure fluctuations lies in their role as a source of structural excitation and reradiation of acoustic noise. We developed a new experimental method for study the dynamics of near-wall turbulence problems [3, 4] is based on the higher moments investigation. The models that most adequately describe the spatial structure of turbulent pressure are the continual statistical models specified by a characteristic functional which provide a complete statistical description of the random field of pressure fluctuations. In this paper, we analyze simple analytical representations of the characteristic functional of turbulent wall-pressure fluctuations.

The functional approach to measuring turbulent pressures provides an almost exhaustive description of the random field on the basis of the experimentally measured characteristic functional of the turbulent fields. A method for the experimental study of the characteristic functional and multidimensional distributions of parameters of the field of turbulent pressure fluctuations is described. Functional approach is based on representation of the statistics of turbulent pressure in the form of an estimate of the characteristical functional. The method combines the analog spatial averaging of turbulent pressure fluctuations over transducer aperture and single-channel statistical processing of signals from pressure transducers with different apertures. The functional approach is tested in the experiment with a wall-bounded turbulent jet flow. The functional approach makes it possible to obtain an exhaustive description of random field based on experimental investigation of characteristic functional of wall-turbulent pressure field. The recent studies of turbulent pressure fluctuations are aimed not only at solving applied problems of noise and vibration generation in boundary-layer flows but also at improving our knowledge of the physical processes that occur in turbulent flows. The main problem is related to the turbulent energy transformation near the flow boundary. However, the physical processes in a turbulent boundary layer are still poorly understood. By now, it is established that, turbulent pressure fluctuations are related to at least two types of near-wall turbulent structures, namely, convective turbulent structures (i.e., free turbulence carried by the mean flow) and turbulence generated by the shear flow formed in the wall region of the turbulent boundary layer. The large-scale motion is represented by accumulations of small-scale structures of different dimensions, intensities, and orientations. In view of the new understanding of the processes of near-wall turbulence generation and dissipation and the role of coherent structures formed in the boundary layer, the problem of the diagnostics of the spatial structure of near-wall turbulence as well as the application of new experimental techniques and the search for more informative characteristics of turbulence attract particular interest. Now, it is evident that, with the use of conventional methods based on the study of statistical moments of the first and second orders, this problem cannot be solved, because the conventional correlation approach leaves out some important properties of turbulence and provides insufficient information on the flow structure. Simple models of the characteristic functional are considered in the context of analyzing the probabilistic characteristics of turbulent pressure fluctuations. The Gaussian model of the spatial characteristic functional of wall-pressure fluctuations is shown to be more appropriate for jet flows, while the Poisson model better describes the characteristic features of near-wall-pressure fluctuations in a turbulent boundary layer. The suggestion is made that the representation of the characteristic functional as a superposition of simple models can reduce the experimental determination of the characteristic functional and the multidimensional distribution functions to measuring only a limited number of parameters and dependences characterizing the turbulent flow under study. In this case, using the simple functional models, one can reduce the experimental determination of the characteristic functional and multidimensional distributions of the turbulent pressure field to the measurement of a limited number of parameters characterizing the turbulent flow under study. Namely, one should measure the spatial correlation function of turbulent pressure fluctuations for the Gaussian field and the some dependences for the Poisson component of the turbulent pressure field.

References

[1] Blake, W.K. Mechanics of flow-induced sound and vibration. V.II. Complex Flow- Structure Interactions. 1986. - N.Y: Academic Press.

[2] Ffowcs, Wiliams J.E. Aeroacoustics. *Sound and Vibr.* **190** (1996), pp. 387- 398.

[3] Kudashev, E.B.; Yablonik, L.R. Experimental method for the assessment of the characteristic functional and multidimensional characteristic functions of turbulent pressure fluctuations. *Acoust. Phys.* **45** (1999), pp. 467–471.

[4] Kudashev, E.B.; Yablonik, L.R. Simple models of the characteristic functional in hydrodynamic acoustics. *Acoust. Phys.* **48** (2002), pp. 321–324.

POINT VORTICES IN A CIRCULAR DOMAIN: STABILITY, RESONANCES, AND INSTABILITY OF STATIONARY ROTATION OF A REGULAR VORTEX POLYGON

LEONID G. KURAKIN

Department of Mechanics and Mathematics, Rostov-on-Don State University, Russia
E-mail: kurakin@math.rsu.ru

The paper is devoted to stability of the stationary rotation of a system of n equal point vortices located at vertices of a regular n-gon of radius R_0 inside a circular domain of radius R with a common center of symmetry. T. X. Havelock stated (1931) that the corresponding linearized system has an exponentially growing solution for $n \geq 7$, and in the case $2 \leq n \leq 6$ – only if parameter $p = R_0^2/R^2$ is greater than a certain critical value: $p_{*n} < p < 1$. In the present paper the problem on stability is studied in exact nonlinear formulation for all other cases $0 < p \leq p_{*n}$, $n = 2, \ldots, 6$. We formulate the necessary and sufficient conditions for $n \neq 5$. For the vortex pentagon it remains unclear the answer to the question about stability for a null set of parameter p. A part of stability conditions is substantiated by the fact that the relative Hamiltonian of the system attains a minimum on the trajectory of a stationary motion of the vortex n-gon. The case when its sign is alternating, arising for $n = 3, 5$, did require a special study. This has been analyzed by the KAM theory methods. Besides, here are listed and investigated all resonances encountered up to forth order. It turned out that two of them lead to instability.

Some results of the present work were briefly reported in [1, 2].

References

[1] Kurakin, L. G. Stability, resonances, and instability of regular vortex polygon in a circular domain. *Dokl. Akad. Nauk* **399** (2004), pp. 52—55, [*Dokl. Phys.* **49** (2004), pp. 658–661.]

[2] Kurakin, Leonid. On stability of a regular vortex polygon in the circular domain. *J. Math. Fluid Mech.* **7** (2005), suppl. 3, pp. S376–S386.

THE PROPAGATION OF STRATIFIED MONOPOLES ON A SPHERE

PETER JAN VAN LEEUWEN

Institute for Marine and Atmospheric Research Utrecht,
Utrecht University, The Netherlands
E-mail: p.j.vanleeuwen@phys.ruu.nl

Expressions are derived for the propagation speed of a monopole in a stratified fluid on a sphere. Numerous publications exist on the evolution of vortices in a one or two-layer fluid on the beta plan. Recently the propagation of point vortices and rotating discs on a sphere have been studied (see e.g. [1, 2, 3, 4]). Van der Toorn [5] was the first to generalize the reduced-gravity vortex motion to the full sphere in his PhD thesis. Ripa [3, 4] studies the same problem in the open literature. The full problem of a rotating body of mass with finite dimension in a stratified fluid has not been tackled before.

By integrating the momentum equations over the sphere equations for the propagation speed of the center of mass of the vortex are obtained. Terms related to precession and nutation are recovered, and the role of extra terms related to the inner motion of the vortex and the stratification is illuminated. Although the expressions are complicated, the meaning of the different terms is clear.

Special emphasis is put on (nearly) steady state propagation. Because the mass anomalies in all fluid layers have to move with the same speed extra conditions on the propagation speed are obtained which allow a solution in terms of simple input variables.

Some time is spent on the propagation mechanism of cyclones, which, because of their negative mass anomaly, are accelerated opposite to the direction of the net force on the mass anomaly.

References

[1] McDonald, N. Robb. The time-dependent behaviour of a spinning disc on a rotating planet: a model for geophysical vortex motion. *Geophys. Astrophys. Fluid Dynam.* **87** (1998), pp. 253–272.

[2] Nycander, J. Analogy between the drift of planetary vortices and the precession of a spinning body. *Plas. Phys. Rep.* **22** (1996), pp. 771–774.

[3] Ripa, P. Effects of the Earth's curvature on the dynamics of isolated objects. Part I: The disk. *J. Phys. Oceanogr.* **30** (2000), pp. 2072–2087.

[4] Ripa, P. Effects of the Earth's curvature on the dynamics of isolated objects. Part II: The uniformly translating vortex. *J. Phys. Oceanogr.* **30** (2000), pp. 2504–2514.

[5] Van der Toorn, R. Geometry, angular momentum and the intrinsic drift of ocean monopolar vortices. *PhD thesis, Utrecht University*, 1997.

TRANSITION TO CHAOS IN THE DYNAMICS OF POINT VORTICES

IVAN S. MAMAEV, ALEXEY V. BORISOV, ALEXANDR A. KILIN

Institute of Computer Sciences, Russia
E-mail: mamaev@ics.org.ru

Transition to chaos in the problem of motion of four point vortices in a plane is considered. A new effective method of order reduction for a system of point vortices in a plane is suggested. For the case of four vortices, the existence a cascade of period-doubling bifurcations is indicated.

This work is supported by the RFBR (Project № 04-05-64367) and INTAS (Project № 04-80-7297).

VORTEX KELVIN MODES WITH NONLINEAR CRITICAL LAYERS

SHERWIN A. MASLOWE, NILIMA NIGAM

Deptartment of Mathematics, McGill University, Canada
E-mail: maslowe@math.mcgill.ca

The propagation of helical perturbations to a columnar vortex seems to have been studied first by Lord Kelvin whose results were published in 1880. In cylindrical coordinates (r, θ, z), one considers the evolution of infinitesimal perturbations $(u_r, u-\theta, u_z)$ superimposed on a flow with velocity profile $\{0, \overline{V}(r), 0\}$. More than 100 years later, the subject remains of considerable interest because of its pertinence to applications in both engineering and geophysical fluid dynamics. A swirling flow is one with velocity components $[0, \overline{V}(r), \overline{W}(r)]$ and the definitive paper treating the linear stability of such flows is by Howard and Gupta [2]. By combining the continuity and three momentum equations, they obtained a single differential equation for the radial perturbation velocity. In order to separate variables, this quantity can be written as $u_r = \varepsilon u(r) sin\xi$, where $\xi \ll 1$ is an amplitude parameter and the phase $\xi = kz + m\theta - \omega t$, k and m being the axial and azimuthal wavenumbers and ω the frequency. Far downstream of an airplane, the axial velocity in the trailing vortices is small so many studies motivated by that application take $W = 0$. The ODE derived by Howard and Gupta can then be written

$$\gamma^2 D\{SD_* u\} - \{\gamma^2 + \frac{m\gamma}{r^2}\left(D[SD(r\overline{V})] - 3\frac{S}{r}D(r\overline{V})\right) - 2\overline{V}k^2\frac{S}{r}Q(r)\}u = 0, \quad (1)$$

where

$$D = \frac{d}{dr}, \quad D_* = \frac{d}{dr} + \frac{1}{r}, \quad \gamma(r) = m\frac{\overline{V}}{r} - \omega, \quad S = \frac{r^2}{m^2 + k^2 r^2} \quad \text{and} \quad Q(r) = \frac{D(r\overline{V})}{r}. \quad (2)$$

In the case of a neutral or weakly amplified mode, critical point singularities occur at any value of r for which $\gamma(r) = 0$. Let us discuss briefly the significance and implications of this singularity. Critical point singularities occur in many shear flows and they are a signal that some neglected effect, usually viscosity or nonlinearity, is important in the neighborhood of the singularity. Because our interest here is in high Reynolds number applications, nonlinearity is the appropriate choice. To deal with the singularity, we therefore introduce a thin critical layer centered on the critical point r_c , in which nonlinear effects are important. The singularity is in the form of an algebraic branch point and series solutions of (1) obtained by the method of Frobenius have a behavior almost identical with those of the Taylor-Goldstein equation governing stratified shear flows. An equivalent Richardson number can even be defined that is proportional to $Q(r)$, the vorticity defined in (2); however, this does not lead to a stability criterion because the analogy between rotating and stratified flows is only valid for axisymmetric disturbances. The mathematical similarities are nonetheless worth noting and we can expect the solutions of the nonlinear vortex critical layer to yield coherent structures analogous to those in a stratified shear flow. The latter bear a strong resemblance to radar observations of large amplitude waves propagating in the atmosphere, as discussed in the review article by the first author [see Fig. 4 in [3]]. More recent work by Troitskaya [5], derives relationships valid at $O(1)$ Richardson number that relate nonlinear effects such as mean flow distortion to changes in the Reynolds stress as the critical layer is crossed. We have employed the same methods that she used to determine changes in the mean velocity and vorticity resulting from the wavemean flow interaction. Essentially, the governing equations are averaged in the ξ direction and then integrated across the critical layer.

The critical layer equations are comprised of a system of four coupled PDEs. The continuity and radial momentum equations are linear, the latter representing a balance between pressure gradient and the linearized centrifugal force. The axial and azimuthal momentum equations, however, are highly nonlinear because of the convective inertial terms. The leading order viscous terms are multiplied by a parameter $\lambda = 1/Re\varepsilon^{3\beta}$, where Re is the Reynolds number and the constant β varies between 1/2 and 2/3. The condition $\lambda \ll 1$ means that the nonlinear critical layer thickness ε^{β} is much greater than that of the viscous critical layer, whose thickness is $Re^{-1/3}$. In most applications outside of the laboratory, that condition will be easily satisfied. In the inviscid $\lambda = 0$ limit, analytical solutions can be obtained using the method of charac teristics. This is most surprising because the system has only two characteristic directions, rather than the four required to be totally hyperbolic. The arbitrary functions in the characteristics solution can be determined by matching above and below the critical layer. However, there is a region of closed characteristics analogous to the Kelvin cat's eyes in the plane wave case. In the 2D case, the vorticity must be constant within a region of closed streamlines according to the PrandtlBatchelor theorem. We were able to extend that theorem enabling us to find the vorticity jump across the critical layer and to match all variables. While the structure of the flow is of interest in its own right, an equally important question relates to solutions of the eigenvalue problem associated with (1). In the case of stratified shear flows, we know that nonlinear critical layers permit the existence of singular neutral modes under conditions where they would be prohibited in a linear theory. This is possible because the jump conditions across the singularity are different thereby permitting new solutions. The existence of such solutions is important in applications such as the vortex interaction between trailing vortices in the aircraft wake problem. Sipp and Jacquin [4], for example, have argued that the dispersion relation for Kelvin modes on a continuous vortex profile is not compatible with the conditions required for the elliptic (or short wave cooperative) instability. Their argument is based on linear viscous stability calculations for a LambOseen vortex. We have shown, however, that the required modes do exist if the critical layer is nonlinear rather than viscous. To conclude, we mention a potential application of the theory that we intend to pursue and that is to hurricanes. A hurricane is a highly axisymmetric rapidly rotating vortex. Radar ob servations show, however, asymmetries such as spiral rainbands that some meteorologists believe are the result of "vortex Rossby waves". The latter are "Rossbylike" waves that are oscillatory in the azimuthal and radial directions. In recent numerical simulations, Chen, Brunet and Yau [1] determined that the vortex Rossby waves have critical layers located about 50 km from the eye of the hurricane (see Fig. 11). These are time dependent and may be responsible for the intensification of the hurricane through momentum absorption of the vortex Rossby waves.

References

[1] Chen, Y.; Brunet, G.; Yau, M. K. Spiral bands in a simulated hurricane. Part II: Wave activity diagnostics. *J. Atmos. Sci.* **60** (2003), pp. 1239-1255.

[2] Howard, L. N.; Gupta, A. S. On the hydrodynamic and hydromagnetic stability of swirling flows. *J. Fluid Mech.* **14** (1962), pp. 463–476.

[3] Maslowe, S. A. Critical layers in shear flows. *Annual Review of Fluid Mechanics*, **18** (1986), 405-432.

[4] Sipp, D.; Jacquin, A. L. Widnall instabilities in vortex pairs. *Phys. Fluids* **15** (2003), pp. 1861-1874.

[5] Troitskaya, Yu. I. The viscousdiffusion nonlinear critical layer in a stratified shear flow. *J. Fluid Mech.* **233** (1991), pp. 25-48.

WEAK TURBULENCE OF SHORT EQUATORIAL WAVES

SERGEI B. MEDVEDEV[1], VLADIMIR ZEITLIN[2]

[1]Istitute of Computational Technologies, Siberian Division of
Russian Academy of Sciences, Russia
E-mail: medvedev@ict.nct.ru
[2]Laboratoire de Météorologie Dynamique CNRS - ENS, France

We derive a normal form of nonlinear equations for short equatorial waves considered in the framework of the rotating shallow water model. We show dynamical splitting of equatorial Rossby and inertia-gravity waves. We derive an effective Hamiltonian for the short inertia-gravity waves and consider their kinetics using the weak turbulence approach. Stationary power- law energy spectra are obtained. They have different slopes for eastward and westward propagating waves due to the fact that resonant triads of inertia-gravity waves exist only in specific regions of the phase-space.

DYNAMICS OF HAIRPIN VORTEX PACKETS IN WALL TURBULENCE

VYACHESLAV V. MELESHKO[1], EUGENE I. NIKIFOROVICH[2],
AlEXANDRE A. GOURJII[2], RONALD J. ADRIAN[3]

[1]Kiev National Taras Shevchenko University, Ukraine
[2]Institute of Hydromechanics, National Academy of Science, Ukraine
[3]University of Illinois at Urbana-Champaign, USA
E-mail: gourjii@i.com.ua

Summary

The talk addresses the experimental, analytical and numerical modelling of the ynamics of concentrated vortex packets over a rigid smooth plane. To answer the principal question why and how does fluid in outer region of the turbulent boundary layer organize itself into hairpin streamwise vortex packets with low-speed convective velocity we developed the vortex filament model of hierarchy of hairpin packets.

Introduction

The coherent structure of the turbulent boundary layer has been studied for about fifty years. Based on a combination of analysis and physical insight, Theodorsen in 1952 proposed a simple vortex model as the central element of the turbulence generation process in shear flows. It took the form of a hairpin (or horseshoe)-shaped vertical structure inclined in the direction of mean shear (see Fig. 1). Since that time a large amount of turbulence structure models has been proposed by numerous investigators, see e.g. [1] for a review. These models typically involve similar single or multiple configurations of hairpin vortical structures. Probably the most convincing evidence of the existence of such vertical structures embedded in fully developed turbulent boundary layers has come from recent experimental [2] and computational (e.g. [3, 4]) studies. The idea is to explore the flow of effectively inviscid fluid with embedded vorticity, with topology change allowed upon close encounters of vortical fluid regions.

Vortex filament ε-model

We consider thin vortex filaments over a flat rigid plane (with no-penetration conditions) embedded into a shear flow of an effectively inviscid fluid. Virtual mirror filaments maintains no-penetration condition. The induced velocity field is defined by means of the Biot-Savart law:

$$U_x = \frac{1}{4\pi} \int\limits_L \Gamma(x', y', z') \frac{(y-y')dz' - (z-z')dy'}{\{(x-x')^2 + (y-y')^2 + (z-z')^2\}^{3/2}} ,$$
$$U_y = \frac{1}{4\pi} \int\limits_L \Gamma(x', y', z') \frac{(z-z')dx' - (x-x')dz'}{\{(x-x')^2 + (y-y')^2 + (z-z')^2\}^{3/2}} ,$$
$$U_z = \frac{1}{4\pi} \int\limits_L \Gamma(x', y', z') \frac{(x-x')dy' - (y-y')dx'}{\{(x-x')^2 + (y-y')^2 + (z-z')^2\}^{3/2}} .$$

The main difference from traditional models is that we consider the strength of the filament dependent on its position. Lagrangian vortex models track fluid elements containing vorticity. The kinematics is governed by geometric relations.

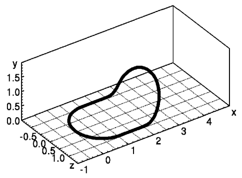

Fig 1. Representation of hairpin vortex by a vortex filament.

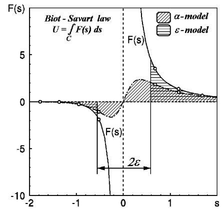

Fig 2. Integration according to the Biot-Savart law.

Advantages of the model

- Numerical integration by the formulas of the various order,
- It accounts the cross-section size of the vortex tube,
- Representation of the vortex tube by a smooth curve,
- Description of vortex tube by small number of markers.

Disadvantages of the model

- Increasing the order of integration methods leads to longer calculation.

Results

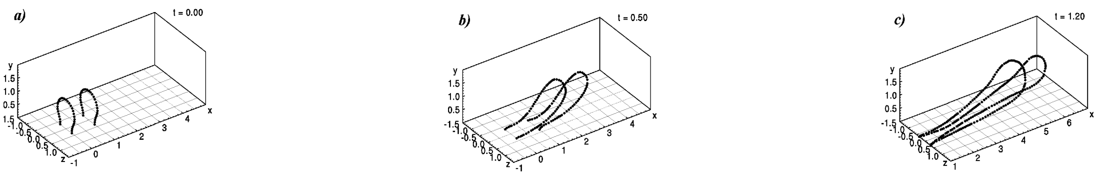

Fig 3. Dynamics of two hairpin vortices above a rigid wall in a shear flow.

Conclusion

We addressed the global vorticity dynamics by representing each hairpin vortex as a filament with a ‘core parameter’, interacting via the Biot-Savart law. The contour kinematic spline method for tracing the vortex filaments in a shear flow over a rigid wall was developed. Special attention is paid to the soliton-like behaviour of the vortex filaments over the rigid plane. Comparisons with experimental results and DNS data show a good correspondence. Although an extreme idealization, the analytical model of vortex filaments appears to shed considerable light on what to expect in the laboratory experiments. The results obtained for the concentrated hairpin vortex structures confirm von Kármán [5] words that "many peculiarities of real flow can be understood based on the notion of existence of separated vortices in the flow and the laws of motion of such vortices in an ideal fluid".

References

[1] Robinson S. K. Coherent motion in the turbulent boundary layer. *Annu. Rev. Fluid Mech* **23** (1991), pp. 601-639.

[2] Adrian, R. J.; Meinhart, C. D.; Tomkins, C. D. Vortex organization in the outer region of the turbulent boundary layer. *J. Fluid Mech.* **422** (2000), pp. 1—54.

[3] Perry A. E., Chong M.S. On the mechanism of wall turbulence. *J. Fluid Mech.* **119** (1982), pp. 173-217.

[4] Spalart P. R. Direct simulation of a turbulent boundary layer up to $Re_\theta = 1410$. *J. Fluid Mech.* **187** (1988), pp. 61-98.

[5] von Kármán Th. Mechanische Ahnlichkeit und Turbulenz. Nachr. Ges. Wissen. Göttingen. *Math-Phys. Kl.* (1930), pp. 17-25.

SOME QUESTIONS OF DYNAMICS OF SUBSTANCE IN THE SPHERICAL VORTEX

ELENA MILYUTE, VALENTINA MILYUVIENE, ALGIMANTAS J. V. MILYUS

The Lentvaris group of European and Lithuanian Physical Societies, France
E-mail: emilyute@mail.ru,

The detailed analysis of the experimental data thus far accumulated on different kinds of interactions allowed us to develop the concept of charge - the basic element of interactions [1, 2, 3]. An appearing of charge is conditioned by the internal mechanism of motion of the liquid viscous mass, forming the spherical vortex. It was found that the charge is fluxes (jets) of particles — volume elements of the mass of the vortex, flowing out from the centers of interacting bodies in radial directions by electrostatic interactions and from out poles of magnets by magnetic interactions. These appearing in the center of the spherical vortex fluxes of substance carry rotation moments, posses kinetic energy and create dynamical or velocity pressure [4, 5, 6], which is responsible not only for formation of stratified layers in the body of the vortex, but for creating of ring vortices - convection cells [3], being the form of existence of radiation outside the vortex itself.

The hydrodynamic model of an inertial structure of interacting objects is presented and mechanisms of formation of various charge-forming fluxes of substance were considered.

References

[1] Miliute, E.; Miliuvene, V.; Milius, A. J. V. The nature of charge, the mechanism of radiation and the inside morphology of nucleus. International Conference "Fluxes and Structures in Fluids", 23—26 June, 2003, St.-Petersburg, Theses of report, pp. 104 — 106.

[2] Miliute, E.; Miliuvene, V. The Dynamics of Substance in a Spherical Vortex and Fields of Interactions. In International EUROMEX 448 Conference "Vortex Dynamics and Field Interactions", September 6-10, 2004, Paris, Theses of report, p. 60.

[3] Milyute, E.A.; Milyuvene, V.A.; Milyus, A.J.V. The nature of charge, the mechanisms of radiation and the internal morphology of interacting objects. *Vilnius.: Moksloaidai*, 2005 (in Russian).

[4] Sedov, L. I. Mechanics of Continuous Media. *Moscow, Nauka*, 1970.

[5] Loytsyanskyi, L. G. Mechanics of Liquid and Gas. *Moscow, Nauka*, 1987.

[6] Saffman, P. G. Vortex Dynamics. *Moscow, Nauchnyi Mir*, 2000.

INFLUENCE OF AN ICE COVER ON EDGE WAVES

SERGEY V. MUZYLEV

P. P. Shirshov Institute of Oceanology, Russian Academy of Sciences, Russia
E-mail: smuzylev@mail.ru

Edge waves on ice-covered water are analyzed in the linearized theory for a plane-sloping beach with a straight coastline. These waves propagate along the coast and have an amplitude which decays exponentially away from the shoreline. The problem is examined without making a hydrostatic assumption. The sea water is considered homogeneous, inviscid, irrotational and incompressible. The ice is taken as of uniform thickness, with constant values of Young's modulus, Poisson's ratio, density and compressive stress in the ice. The boundary conditions are such that the normal velocity at the bottom is zero and at the undersurface of the ice the linearized kinematic and dynamic boundary conditions are satisfied. We present and analyze explicit solutions for the edge flexural-gravity waves and the dispersion equations.

This study is supported by the grants from RFBR 05-05-64212 and 06-05-65210.

TRIPLET OF HELICAL VORTICES

VALERIY L. OKULOV[1,2], IGOR V. NAUMOV[1], WEN Z. SHEN[2], JENS N. SORENSEN[2]

[1]Institute of Thermophysics, Siberian Division of Russian Academy of Sciences, Russia
[2]Department of Mechanical Engineering, Technical University of Denmark, Denmark
E-mail: vokulov@mail.ru

Results of laboratory search and numerical identification of a stable triplet of helical vortices embedded in a strong swirling flow are presented. As basic flow configuration, swirling flows induced by a rotating cover in a closed cylinder have been considered. The main idea of the study is to determine the essential influence of an assigned flow on the stability of helical multiples, that has previously been established theoretically [1]. From the theoretical study it was shown that the stability zone of a helical multiple embedded in a swirl flow essentially extends to very small values of the helical pitch when the strength of the assigned axisymmetric flow field is increased (Fig. 1).

The experimental investigation was carried using simultaneously two diagnostics methods: Particle Image Velocimetry (PIV) to determine velocity fields by particle tracks and Laser Doppler Anemometry (LDA) to establish time-histories [2]. The swirling flow in the cavity was measured in a horizontal cross-section of the cylinder. The flow was systematically investigated for flow structures and, as a result, the existence of a stable vortex triplet was found at operating conditions corresponding to those predicted by the theoretical stability analysis (Fig. 1). Fig. 2 shows the measured velocity field induced by the vortex triplet after separation of the axisymmetric flow field.

For the same flow regime the 3D unsteady Navier-Stokes equations were solved numerically to identify the 3D structure of the vortex triplet. As a result, the existence of steady vortex triplets with a strongly pronounced helical structure has now been determined both experimentally and

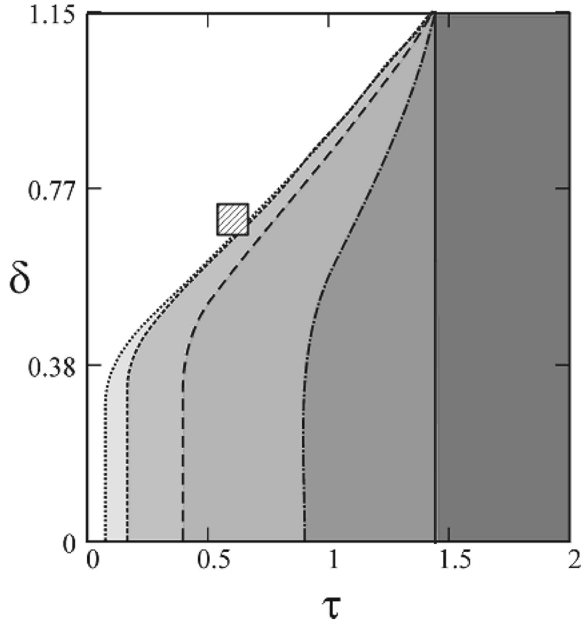

Fig 1. Stability regions for helical triplets embedded in a assigned flow for different values of the circulation ratio 0 (———); 0.5 (− − − −); 5 (- . - . - . - .); 50 (- - - - - -); 500 (. . . . .) as function of helical pitch and , the ratio of the assigned vortex radius to the triplet radius. The circulation ratio designates the ratio between the total circulation of the triplet and the vortex in the assigned flow. Stable regions are located on the side of the curve with the most intensive color and the crosshatched square indicates flow characteristics for the regimes in which a helical vortex triplet has been detected in the lid-driven rotating cavity.

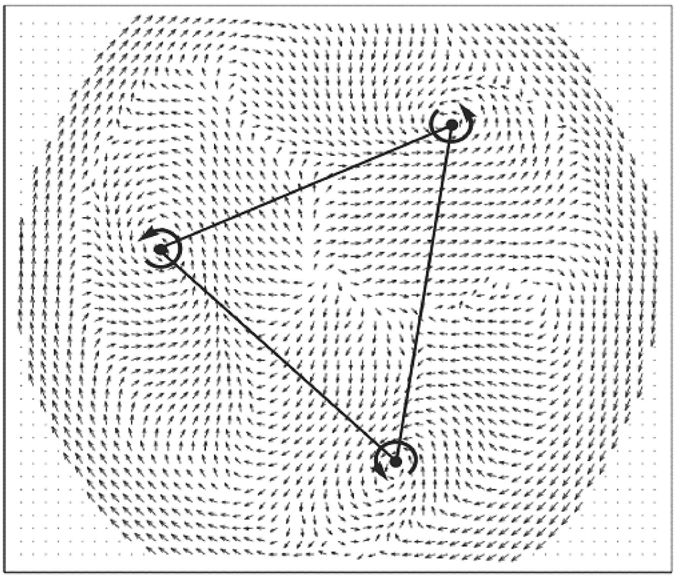

Fig 2. Measured velocity field induced by vortex triplet in a horizontal cross-section.

numerically. There exist several investigations of flows in the lid-driven rotating cavity (e.g., [3, 4]) in which various types of periodic multiplicity disturbances of the axisymmetric vortex field were detected both numerically and experimentally. However, in the present study the stability of a helical vortex triplet has been predicted theoretically for the first time, and its existence proven experimentally as well as numerically.

References

[1] Okulov, V.L.; Sorensen, J.N. Stability of helical tip vortices in rotor far wake. Submitted to *J. Fluid Mech.*

[2] Naumov, I.V.; Okulov, V.L.; Mayer, K.E.; Sorensen, J.N.; Shen, W.Z. LDA - PIV diagnostics and 3D simulation of oscillating swirl flow in a closed cylindrical container. *Thermophysics and Aeromechanics*, **10** (2003), pp. 143–148.

[3] Gelfgat, A.Y.; Bar-Yoseph, P.Z.; Solan, A. Three-dimensional instability of axisymmetric flow in rotating lid-cylinder enclosure. *J Fluid Mech.* **438**, (2001), pp. 363–377.

[4] Lopez, J.M.; Marques, F.; Hirsa, A.H..; Miraghaie, R. Symmetry breaking in free-surface cylinder flow. *J Fluid Mech.* **502** (2004), pp. 99–126.

FAMILIES OF TRANSLATING NEUTRAL VORTEX STREET CONFIGURATIONS

KEVIN A. O'NEIL

Deptartment of Mathematics, The University of Tulsa, USA
E-mail: koneil@utulsa.edu

A simple but useful model of the 2D wake of a bluff body is a periodic arrangement of point vortices in the plane, with circulations and positions adjusted so that the total circulation is zero and all vortices have the same velocity. A complete account of the cases of two and three vortices per fundamental strip may be found in [1]; examples and many other references are collected in [2].

A fruitful method for constructing stationary *non*–periodic vortex configurations is to place vortices of circulations +1/-1 or +1/-2 at the roots of polynomial solutions to a certain bilinear differential equation of hypergeometric type [3, 4, 5]. In the present work this technique is adapted to allow the construction of translating neutral vortex street configurations. A multilinear ordinary differential equation is derived that corresponds to the periodic case with arbitrary vortex circulations. Families of solutions for +1/-k systems are found as well as for systems with more than two distinct circulations. Also, a construction in [5] that generated stationary non-neutral vortex street configurations is adapted to find solution families that are neutral and have arbitrary common velocity. Some of these solution families are particularly general in that they contain continuous parameters other than the common velocity.

References

[1] Stremler, Mark A. Relative equilibria of singly periodic point vortex arrays. *Phys. Fluids* **15** (2003), pp. 3767–3775.

[2] Aref, H.; Newton, P.K.; Stremler, M.; Tokieda, T.; Vainchtein, D. Vortex crystals. *Adv. Appl. Mech.* **39**(2003), pp. 1–79.

[3] Bartman, A. B. A new interpretation of the Adler-Moser KdV polynomials: interaction of vortices. *Nonlinear and turbulent processes in physics, Vol. 3 (Kiev, 1983)*, pp. 1175–1181, *Harwood Academic Publ., Chur*, 1984.

[4] Loutsenko, Igor. Equilibrium of charges and differential equations solved by polynomials. *J. Phys. A* **37** (2004), pp. 1309–1321.

[5] Loutsenko, Igor. Integrable dynamics of charges related to the bilinear hypergeometric equation. *Comm. Math. Phys.* **242** (2003), pp. 251–275.