VORTEX GENERATION BY DENSE WATER OVERFLOWS ON A INCLINE SLOPE

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Dense overflows on a continental slope play an important role in renewing deep water as part of the global thermohaline convective cycle of the oceans. Cold water from Nordic seas or salty water from the Mediterranean Sea descent into the Atlantic ocean over a sill, resulting in mass exchange with high density gradient, instabilities and the generation of mesoscale vortices. The footprint ot these vortices on the ocean surface can be observed from satellites. We discuss here laboratory experiments reproducing this phenomenon. They are performed on the large 'Coriolis' turntable in Grenoble, with particular emphasis on the dynamics of these large vortices.

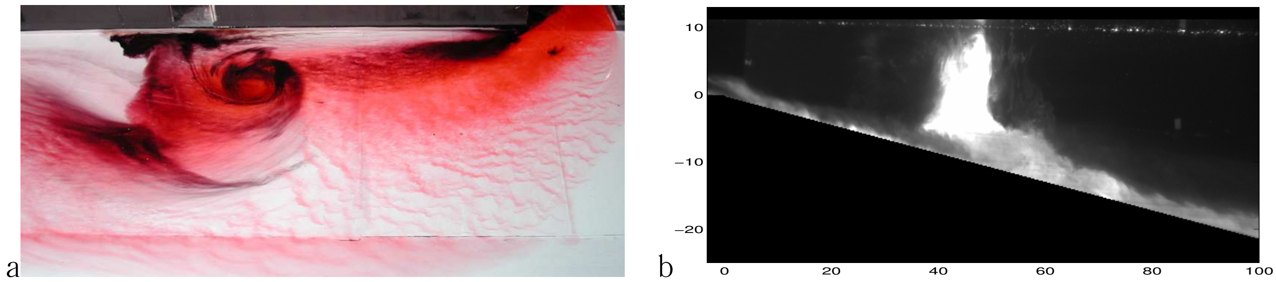

Fig 1. Visualisation of the gravity current behaviour. **a.** The current (red dye) splits into a geostrophically balanced jet flowing along the slope and a thin layer slowed down by viscosity descending the slope. The main current is subject to baroclinic instabilities which induce cyclonic vortices over the gravity current seen by injection of black dye. **b.** The vertical cut (fluorescine illuminated by a vertical laser sheet) shows that the current thickness decreases at the beginning of the slope before growing because of turbulent mixing.

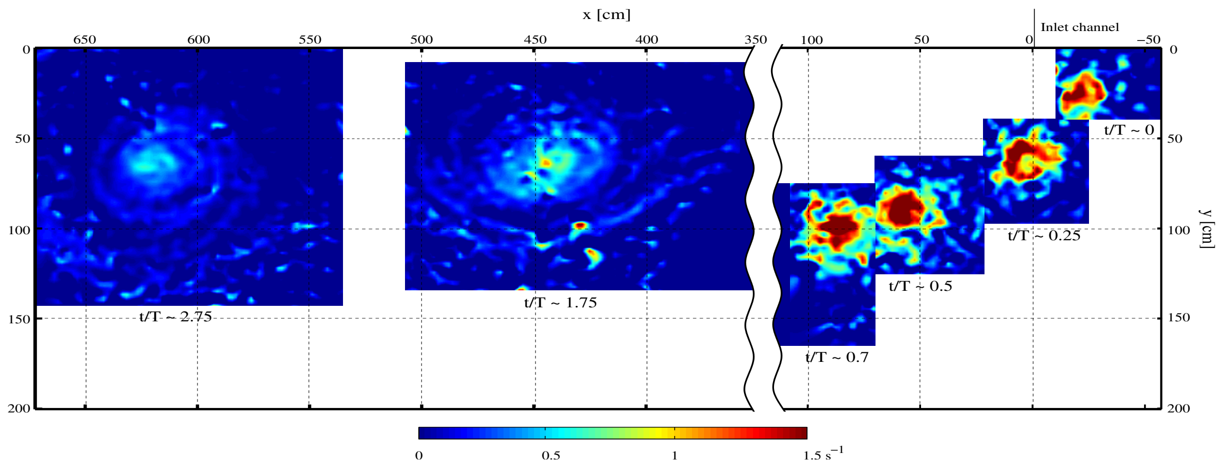

Fig 2. Patchwork showing the evolution with time of the vorticity field in a cyclonic vortex along the slope. The fields are obtained by differentiation of the velocity fields obtained by PIV in a plane at 8 cm from the sloping bottom

The gravity current is created by salty water injected with a constant flux, which then flows down the incline slope (2m wide, 10m long, 15^o of inclination) with intense turbulent mixing occurring at the interface. It is deviated by the Coriolis force to a cross-slope horizontal direction. The ambient fluid is either homogeneous either uniformly stratified in density. In the latter case, the density of the source is that of the bottom water. The dimensions of the experiment permit to obtain a gravity current both turbulent and strongly influenced by rotation, and allow a good similarity with the oceanic scale with Rossby and Burger numbers conserved. For instance a continental slope of 122km wide and 3km deep can be reproduced. Earlier experiments were done either without rotation, or in a quasi-laminar regime.

Velocity fields are measured by particle image velocimetry (PIV). The current is seeded with particles illuminated by a laser sheet directed along the slope at different heights. Probes are also used to record density variations at different locations.

The coherent cyclonic vortices are generated in the inlet channel with a well defined shedding period, equal to twice the tank rotation period. The instability at the origin of the vortices will be discussed. Vortices are further amplified as the current reaches its horizontal cross-wise direction. A mature vortex can be seen in figure 1, as a top view (left) and as a vertical cut (right), in the case of a homogeneous ambient fluid. The barotropic nature of the vortex is apparent in both views. In figure 1a, the black dye is indeed introduced above the vortex while the red dye is introduced in the current source: both dyes follow the same path around the vortex. In figure 1b, fluoresceine is introduced in the current source, and is observed to rise in the vortex core up to the free surface. In the case of a linearly stratified ambient fluid, vortices are similarly generated, and extend above the current, although they are screened by stratification so they decay with height.

The vortex structure has been studied in detail from PIV measurements, from which a succession of vorticity can be obtained, as shown in figure 2. The evolution of vorticity and vortex diameter can be estimated from these fields, as well as the vortex trajectory. The motion results from a competition between advection by the barotropic component of the gravity current and cross-wise horizontal propagation by the topographic beta effect.

In conclusion, the dynamics of these coherent vortices generated by gravity current is an interesting example of interplay between the small scale turbulent motion and meso-scale features. In the oceanic case, a good understanding of their dynamics should allow us to deduce the properties of the deep current from surface observations of the vortices. Further details can be obtained in [1] (availlable on http://www.coriolis-legi.org).

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CLUSTERING OF FLOATERS BY WAVES

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How do waves affect the distribution of small particles floating on a liquid surface? If we assume that small particles move with the fluid, then they just oscillate back and forth in a standing wave, while undergo a (Stokes) drift in the direction of wave propagation in a running wave. Superposition of drifts in a random set of waves is believed to lead to the diffusion of particles [1]. We discovered, however, that for small particles the capillary force may be important making the mass of the displaced liquid not equal to the particle mass [2]. Here we present experimental results and a theoretical model demonstrating that these inertial effects (mass mismatch) produce drift and clustering of particles in a pure standing wave. We also measured the statistics of particle distribution in a set of random waves and showed that particles tend to concentrate on a multi-fractal set.

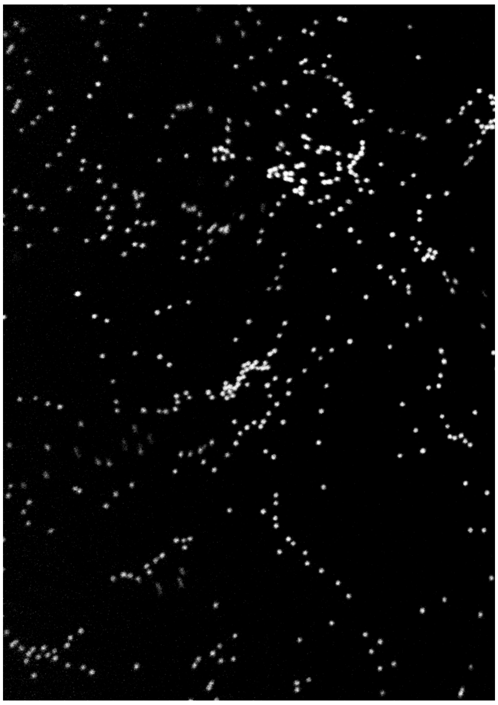
For small floating particles of sizes less then the capillary length the surface tension force becomes comparable with the particle weight. This capillary force pulls hydrophilic particles deeper into the liquid, so the mass of displaced liquid exceeds the particle mass. The mass of hydrophobic particle is greater then that of the liquid it displaces. Indeed, we have observed that on a static inclined surface particles will climb up or slide down depending on their hydrophobic/ hydrophilic properties.

In a standing wave, where the averaged slope is zero, we found that the effect of inertia due to surface tension also leads to particle drift. We described two mechanisms acting in the same direction and providing clustering of hydrophilic/hydrophpbic particles at nodes or antinodes correspondingly. One mechanism, kinematic, is based on the asymmetry of the particle motion with the liquid (the surface slope is steeper during the half-period when the surface is lower than the undisturbed level). The second mechanism takes into account a relative motion of the particle with respect to the liquid (a vertical displacement of inertial floater moving with the wave). This displacement produces an extra restoring force which pushes a hydrophobic particle up from its low position and down from its higher position, and thus provides a non-zero averaged force towards the anti-node. We analyzed both mechanisms in terms of the nonlinear dynamic system and found that the drift appears in the second order with respect to the wave amplitude.

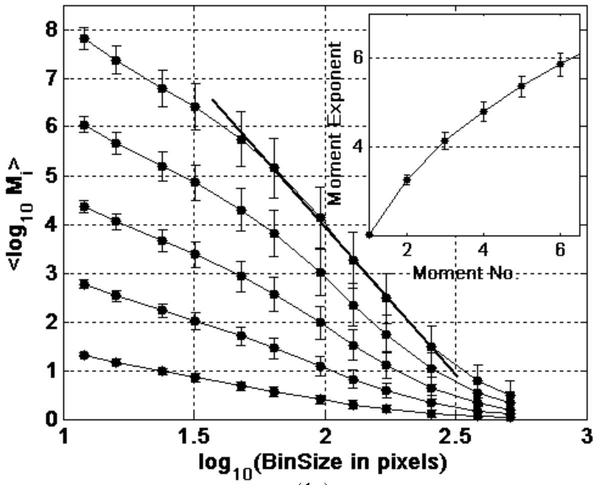
As it was shown by E. Ott [3] chaotic flow generated on a flat liquid surface produces fractal distribution of passive tracers. It is a result of surface flow compressibility which makes chaotic mapping of the surface into itself dissipative. A recent theory for inertial particles in two dimensional compressible flow [4] predicts that particle mass distribution must be actually multi-fractal i.e. the scaling exponents of the density moments do not grow linearly with the order of the moment. We proved this experimentally for inertial particle moving in a surface flow generated by a set of random waves.

The experiments have been done with small hydrophilic or hydrophobic particles floating on a surface of liquid with capillary waves excited. The waves were generated through parametric instability in a vertically vibrated small cell filled with liquid. The cells size was 5x5x1 cm and 10x1x1 cm. The cell was illuminated from below by expanded collimated laser beam. Most of experiments were done with suspension of hydrophilic particles - hollow glass spheres or polymer fluorescent microspheres of sizes 30-90 μm and with particle-liquid density ratio 0.6 - 0.8. Particle distribution and wave pattern were visualized and recorded simultaneously using fluorescent technique and confocal optical system arrangement.

We did two sets of experiments with particle clustering on the surface wave. In the first set with the standing wave, we have shown that the particle drift towards the nodes or anti-nodes depends on the sign of surface tension forces. We measured the clustering time as a function of wave amplitude at different wavelengths from 5 to 12 mm. This experimental results confirms the conclusion made from analytical modelling that clustering time is proportional to the second degree of the wave amplitude. In the second set of experiment we studied statistics of particle distribution in a set of random waves. At large vibration amplitude the capillary wave pattern becomes chaotic, the particles are dispersed but their instantaneous distribution is highly inhomogeneous on the scale less than wavelength(see Fig 3a). We have found that the probability distribution function (PDF) of the particle concentration has essentially non-gaussian shape with exponential tails at large concentration. Fig. 3b shows a growth of concentration moments averaged over bins of sizes from 8x8 to 256x256 pixels (1 pixel = 0.015 mm, the wavelength is 7 mm) for the sizes of the bins less than the wavelength of parametrically excited mode. The scaling exponents for the moments (shown in the inset if Fig. 3b) depends nonlinearly on the number of the moment m, that is the first experimental demonstration of multifractality in the distribution of particles.



(a)



(b)

Fig 3. (a) The image of particle distribution in random waves. Image size 20x20 mm, the wavelength 7 mm. (b) Moments of concentration (1,2,3,4,5 and 6th) versus the scale of coarse-graining. Inset: scaling exponents.

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STEADY VISCOUS FLOWS AND TENSOR INVARIANTS OF DYNAMICAL SYSTEMS

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In [1], N. V. Denisova and V. V. Kozlov develop a constructive method which is then used to introduce the new concepts of chaos and chaotic behavior in regards to stationary currents of an abstract continuous medium. The technique consists in expanding the movement equations of the medium as a power series in degrees of a small parameter and applying the resonant tora destruction conditions, as perturbation gets imposed on the system.

In present talk, two models will be considered: the barotropic compressible viscous fluid and the incompressible ideal fluid with the viscous friction. It is proved, with the help of the Poincaré-Kozlov theorem [2], that the typical stationary flows of the fluid (in the above-mentioned models) have no non-trivial integrals, fields of symmetries and integral invariants. In other words, these flows are chaotic.

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A THEORY FOR INTERNAL GRAVITATIONAL WAVES WITH TRAPPED RECIRCULATION CORE IN DEEP FLUIDS

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This work is concerned with a new theoretical model for long large amplitude internal gravitational waves with a vortex core propagating in a narrow layer of stratified fluid embedded in an infinitely deep homogeneous fluid. Trapped zones of recirculating fluid play an important role in the mesoscale mass transfer and being subject of extensive numerical studies during last decades. Theoretical approach however is still limited to the small but finite amplitude theory leading to the well known Benjamin-Ono type equation for wave amplitude, which comprises quadratic nonlinearity and the integral dispersion. Recently Derzho and Velarde (Phys. Fluids, 7(6), 1995) has shown that for long waves in such environment the small amplitude restriction can be relaxed for any nearly uniform profile of stratification. Thus wave profiles (and other wave properties such as wave speed, etc) can be can be calculated up to a critical amplitude for which wave breaking occurs and thus closed circulation. This new theory adds complicated nonlinearity to the resulting equation on wave amplitude, the explicit form of the nonlinearity depends on the undisturbed density profiles.

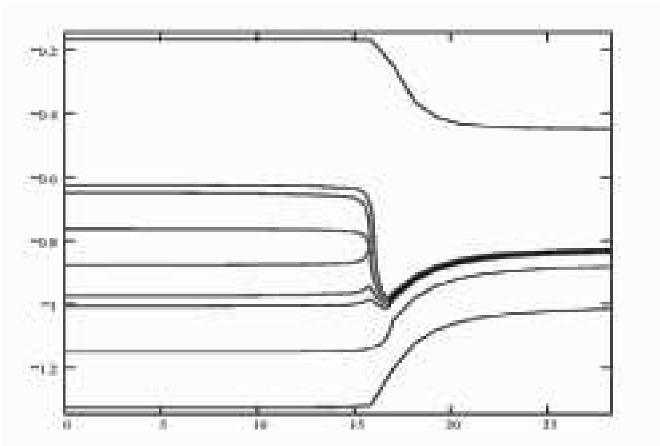

Fig 1

Here we present a model for waves with amplitude slightly greater than the critical amplitude for which breaking occurs. We incorporate a vortex core located near the above point at which breaking occurs and show how to match the recirculating flow inside the vortex with the outer flow using the technique developed earlier (Derzho and Grimshaw, Phys. Fluids, 9(11), 1997; Derzho and Grimshaw, Stud. Appl. Maths, 115, 2005). The effect of the vortex core is to introduce into the governing equation for wave amplitude an extra nonlinear term proportional to the 3/2 power of the difference between the wave amplitude and the critical amplitude. Thus the derived new equation for wave amplitude incorporates the nonlinearity arising due to the flow over the recirculation core and the nonlinearity associated with the ambient stratification, dispersion term however remains of the Benjamin-Ono integral type. We find that as wave amplitude increases above the critical amplitude, the wave broadens, which is in marked contrast to the case of small amplitude waves where sharpening of the wave crest normally occurs. Further, the wave speed is found to depend nonlinearly on the wave amplitude, again with marked contrast to the linear dependence for small amplitude waves. The limiting form of the broadening wave is "a deep fluid bore". It is worth to mention that such wave structure has never been theoretically described before as the Benjamin--Ono model does not allow bore like solutions. Streamlines pattern for "a deep fluid bore" is presented in the figure below.

TURBULENCE AS SECOND ORDER PHASE TRANSITION

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We suggest the study of turbulence as a phase transition of the second order, where the phases are the laminar and turbulent flow by means a suitable system realized by the Ginzburg-Landau and Navier-Stokes equations

HAMILTON-JACOBI EQUATION AND TRAVELLING WAVES INVOLVING ANOMALOUS TRANSPORT AND NON-MARKOVIAN RANDOM PROCESSES

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We present a geometric approach to the problem of propagating fronts into an unstable state, valid for an arbitrary continuous-time random walk with KPP growth rate. We derive an integral Hamilton-Jacobi type equation for the action functional determining the position of reaction front and its speed. Our method does not rely on the explicit derivation of a differential equation for the density of particles. By using the Hamilton equations, we obtain an explicit formula for the propagation speed for the case of anomalous transport involving non-Markovian random processes.

HAMILTONIAN VERSIONS OF CONTOUR DYNAMICS

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This report pursues two aims. The first is to show how the Hamiltonian approach can be used to develop effective analytical versions of contour dynamics. The second is to use Hamiltonian contour-dynamics methods to study of strongly nonlinear vortex structures in models of geophysical fluid dynamics and plasma.

Using the Euler description as a starting point, we present a systematic procedure to reduce two-dimensional patches dynamics of constant vorticity and density to the Hamiltonian dynamics of their contours. Hamiltonian reformulations of this kind depend on the contour parameterizations, which are viewed as constraints and hence play a key role. The special method similar to the Dirac's procedure [1] is used to free the Hamiltonian formulation from the constraints. It is shown that in the weak curvature approximation the main contribution to Dirac's Hamiltonian is defined solely by the constraint, which has a simple topological meaning and expresses a closure condition for the contour. This fact leads to a universality of the theory for all models in the first order of the perturbation theory. It must be emphasized that the perturbation theory is so arranged that approximations do not touch on Poisson brackets in any order. That is why internal symmetry properties remain intact.

Vortex models of geophysical fluid dynamics and plasma are considered as illustrations of possible applications. Among them are quasi–geostrophic and plasma models as well as the cyclostrophic vortex model used for the study of the Venusian hot spots. Vortex solutions of these models have much in common and look like multi–petal structures rotating with constant velocities. Of special interest are two-petal solutions with a self-contacting of the contour in the center of rotation. Analysis shows that vortices of this extreme type minimize the system energy. For this reason, they are most probable pretenders to the role of asymptotic states. In real open low-viscous systems, these states can manifest itself on intermediate stages of the evolution as a universal response to external actions.

A additional information about features of the method and its possible applications can be found in the works [2] – [5].

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DIAPYCNAL MIXING IN TAYLOR-COUETTE FLOW

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We consider the mixing of a two-layer fluid in a Taylor Couette flow. This flow allows establishing well-controlled flow regimes, i.e. Taylor vortices, wavy-vortex and turbulent flow regimes, for which the interfacial mixing is measured quantitatively using the LIF method. Even though flow conditions are different in each regime, the mixing efficiency represented by the flux

Richardson number, Ri_f, as a function of the overall Richardson number, Ri_o, reveals a similar behaviour with an increase in mixing efficiency at small Ri_o up to a maximum value of ≈ 0.10 to 0.25 and then a decrease at large Ri_o. The maximum is reached at $Ri_o \approx 2$ to 10 depending on flow regime. The large spread in Ri_f - Ri_o values is typical of mixing efficiencies reported previously for different types of stirring, including grid stirring and bar stirring experiment. What is of interest here is that the spread is obtained in the same apparatus and is shown to be dependent on Reynolds number. The other interesting observation is that for large Ri_o (here of order 10^2) the mixing efficiency increases again. Since shear instabilities are unlikely to occur for these Ri_o numbers other mixing processes should be responsible. Observations suggest trapping effects that lead to wave breaking at the interface.

The mixing of the upper ocean layer, such as due to internal wave breaking, shear instability, Langmuir vortices, as well as convection e.g. due to evaporation of salt water (see Thorpe, 2004), is a long standing issue in fluid mechanics. Former investigations motivated by this application report isotropic grid-generated turbulence or locally generated turbulence in a (non-rotating) stratified fluid to investigate the mixing efficiency. The Taylor-Couette flow we here consider is anisotropic, consist of annular ring-shaped vortex structures with horizontal axis, and, unlike most other experiments and simulations on mixing allows for the generation of inertial-gravity waves.

Two different Taylor-Couette devices are employed, one of the classical type with a thin gap of $1cm$ and height to gap-width ratio of 52.4 inner diameter of $10cm$ (small TC-device), and a more unconventional one with gap-width $5cm$ of ratio 12.2 and inner diameter of $25cm$ (large TC device) on which we here focus. Since the most unstable wavelength depend on gap-width and Reynolds number, the Taylor vortices in this device will have a larger diameter. Typically, the effect of a linear stratification is to increase the threshold value for instability, or transition to another regime, and to decrease the aspect ratio of the Taylor vortices (see Caton, Janiaud & Hopfinger, 2000). Under the influence of the buoyancy force, the Taylor vortices have a smaller aspect ratio, unlike the wavelength of the waves, generated by centrifugal instability near the inner cylinder wall, that is not affected by stratification. As a consequence, beyond the instability threshold, waves and Taylor vortices appear simultaneously at different spatial scales. In the two-layer fluid with vertically varying density-gradient inertial waves propagate away from the inner cylinder. These propagating waves (see Ermanyuk & Flór, 2005) encounter the shear between the Taylor vortices and the motion at the interface. While their frequency is affected by the local fluid velocity they encounter critical layers and break, are absorbed or reflect above and below the interface where the wave frequency approaches the local buoyancy frequency.

Figure 1 shows the mixing efficiency represented by the Richardson flux number, $Ri_f = \frac{D}{L} E Ri_o$ as a function of the overall Richardson number $Ri_o = \frac{g\Delta\rho}{\bar{\rho}} \frac{L}{U^2}$, obtained for the turbulent regime, with E the nondimensional entrainment rate, D the upper layer height, L and U the vortex diameter and maximum velocity, respectively.

For larger Ri_o, the mixing efficiency systematically increases. Though former studies suggest the influence of waves interacting with the shear layer as a possible mechanism to increase the mixing efficiency (see Strang & Fernando 2001, Peltier & Caulfield, 2003), this has not been shown for such high Ri_o numbers. Observations reveal a mixing layer of approximately 1 to 1.5 cm thickness (see figure), with wave breaking fluid motions, while vortices above and below erase filaments of dense (light) fluid from the upward (downward) moving layer induced by wave motions. The flow represented by the monotonic decreasing part (for smaller Ri_o) of the curve also showed filamentation generated by the same process, but the waves do not break in the interior of the pycnocline. This suggests that the increase in mixing efficiency is a consequence of waves being trapped at the interface and break, thus contributing to the mixing efficiency.

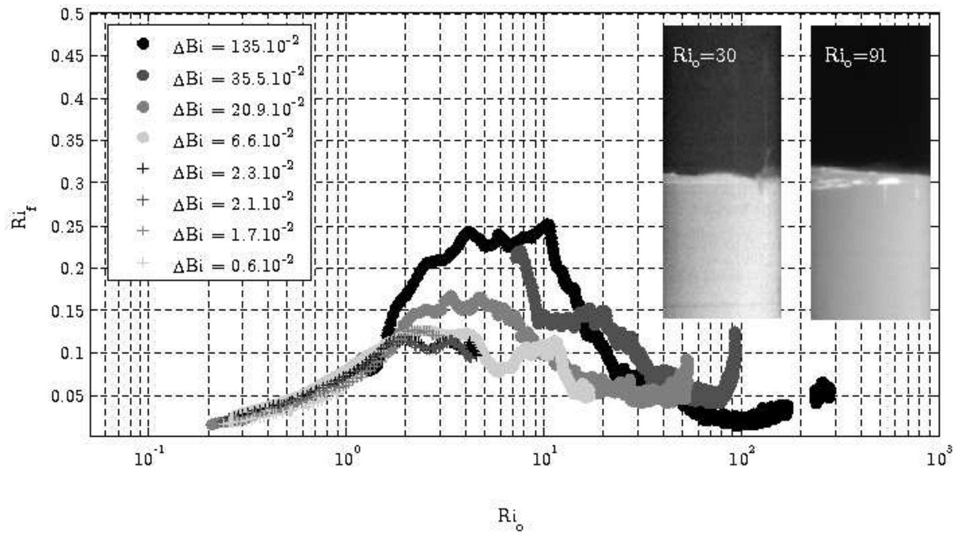

Fig 1. Mixing efficiency Ri_f of the TC flow as a function of the overall Ri-number Ri_o, in the turbulent regime for a constant Reynolds number $Re = \Omega ad/\nu = 3409$ (with cylinder rotation frequency Ω and radius a, d gapwidth) for various $\Delta B = g\Delta\rho/\overline{\rho}$ in m/s^2 (see legend). The temporal evolution of the flow goes from left to right with an initially strong interface (high Ri_o) at the right. The two pictures are for $\Delta B = 35.5\,10^{-2}\,m/s^2$.

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BEACH ZONE VORTICES NEAR STEPPED TOPOGRAPHY

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The motion of ideal shallow water vortices near rectilinear stepped topography is studied without background rotation. Firstly, finite area monopolar vortices which propagate steadily parallel to the step, without change of shape are found. Solutions are obtained numerically and are unique for a given volume and centre of vorticity. Time-dependent integrations show that these vortices are robust: flows initialized with a V-state remain close to the V-state and flows initialized with a circular vortex shed vorticity to approach a V-state. The translational velocity of the vortices is shown to be finite and, unlike that of a singular line vortex, not to increase without limit as the centre of vorticity approaches the escarpment.

Second, the motion of a pair of line vortices is found using Hamiltonian techniques with similar trajectories found for vortex patches using contour dynamics. A comparison between the two trajectories is found to be close provided vortex patch centroids are sufficiently far away from the escarpment. For given constants of motion (energy, linear impulse and circulation) the path each vortex is found to be unique in relative distance variables (up to translations in the x-plane). Furthermore, for special values of the constants of motion, vortex pairs which propagate steadily parallel to the escarpment without deformation (dipole equilibrium states) exist even when the circulation of each vortex has the same sign.

TRANSITION TO INSTABILITY AND REVERSIBLE BIFURCATIONS OF FLOWS WITH PHASE TRANSITION IN HORISONTALLY EXTENDED DOMAINS OF A POROUS MEDIUM

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Bifurcations from steady state, taking place in reversible media (invariant under reflection of spatial coordinates) arise when the eigenvalues of the linearization about the steady state of right hand side of dynamical system, describing the respective stationary problem, come to imaginary axis in pairs. In dispersive fluid mechanics the respective stationary problem corresponds to determination of forms of travelling wave structures, bifurcating from a uniform steady state. In dissipative fluid mechanics the stationary problem corresponds to determination of forms of non-uniform secondary steady wave structures bifurcating from a uniform steady state at the threshold of instability. There are three more or less general types of reversible bifurcations when the complete analytic description of bifurcating structures of small amplitudes are possible via reduction of order of respective dynamical system combined with a normal form analysis of a flow on the centre manifold.

In the frame of dissipative fluid mechanics we consider the stability of vertical flows with phase transitions in horisontally infinite domains of a porous medium. The analysis covers first high-temperature geothermal reservoirs in porous media, consisting of two high-permeability layers, which are separated by a low-permeability stratum. The thermodynamic conditions are assumed to imply that the upper and lower high-permeability layers are filled in by water and by vapor, respectively. In these circumstances the low-permeability stratum possesses the phase transition interface, separating domains occupied by water and vapor. The stable stationary regimes of vertical phase flow between water and vapor layers in the low-permeability stratum may exist. Stability of such regimes where the heavier fluid is located over the lighter one is supported by a heat transfer, caused by a temperature gradient in the Earth's interior. We give the classification of the possible types of transition to instability of the vertical flows in such a system under the condition of smallness of the advective heat transfer in comparison with the conductive one. It is found that in the non-degenerate case there exist three different scenarios of the onset of instability of the stationary vertical phase transition flows.

The other physical situation corresponds to the Rayleigh-Taylor instability of a water layer located over a air-vapor layer in a porous media under isothermal condition in presence of capillary forces at the phase transition interface. The model in question describes, for example, the convective and filtration processes in mines, tunnels and other constructions, having contact with unbounded or semi-bounded natural massifs. The functioning of such systems is accompanied by non-stationary heat and mass exchange between the construction and rock. Artificial ventilation makes it possible to keep the micro-climate, necessary for exploitation. The ventilation is accompanied by evaporation from the ceiling of a construction and water from a water stratum above the mine moves downward under action of gravitational force and a pressure in the water stratum.

In all cases considered the criteria of stability were determined. It was also found that in physically important cases the transition is accompanied by the bifurcations of the destabilizing vertical flow, leading to appearance of horizontally non-homogeneous regimes with non-constant shape of the interface. The bifurcations correspond to the bifurcations of the simple resonance and 1 : 1-resonance, which typically arise in reversible systems and belong to the prescribed types of bifurcations. Moreover, in some cases the bypass transition takes place, when the basic vertical regime ceases to exist as a solution of the governing system of equations above the threshold of instability .

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THE HAMILTONIAN DYNAMICS OF VORTICES IN IRREGULAR, MULTIPLY CONNECTED DOMAINS AND THEIR USING IN PREDICTING FINITE-AREA VORTEX BEHAVIOUR

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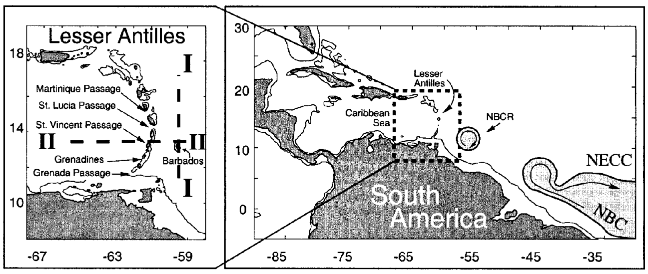

Fig 1. The North Brazil Current (NBC) retroflecting to form the North Equatorial Countercurrent (NECC), and a North Brazil Current ring (NBCR) in the vicinity of the Lesser Antilles. Cross sections I-I and II-II are shown in Figure .

Two models are presented for the motion of vortices near gaps in infinitely long barriers: a problem of geophysical significance. The first model considers a line vortex for which the exact nonlinear trajectories satisfying the governing two-dimensional Euler equations are obtained analytically using conformal mapping results for vortex Hamiltonians. The second model considers a finite area patch of constant vorticity and is based on conformal mappings and the numerical method of contour surgery. The two models enable a comparison of the trajectories of line vortices and vortex patches.

The case of a double gap formed by an island lying between two headlands is considered in detail. It is noted that Kelvin's theorem constrains the circulation around the island to be a constant and thus forces a time-dependent volume flux between the islands and the headlands.

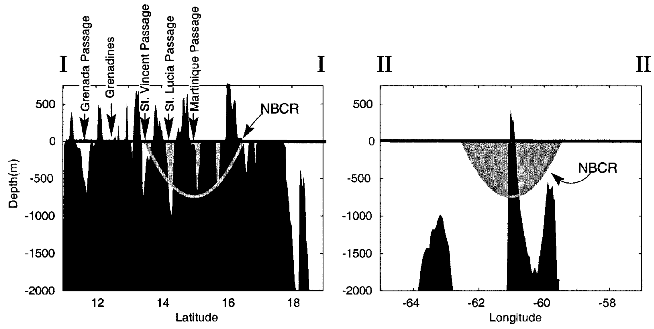

Fig 2. Meridional cross section (left) of the Lesser Antilles from the point of view of a “typical” NBCR approaching the Lesser Antilles from the east. The ring is a “hypothetical” ring superimposed on the topography to illustrate the scales. The location of the section is marked by I-I in Figure . A zonal cross section (right) through the St. Vincent Passage, looking north to St. Vincent Island. The location of this cross section is marked by II-II in Figure .

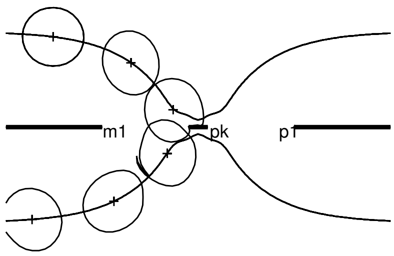

Fig 3. A patch strikes an island and is sufficiently distorted from circular for its centroid to leave the line vortex path before the patch resymmetrises and the centroid returns to a different Hamiltonian isoline still however corresponding to the initial patch energy.

When the gap between the island and a headland is small this flux requires arbitrarily large flow speeds through the gap. In most examples the centroid of the patch is constrained to follow closely the trajectory of a line vortex of the same circulation. Exceptions occur when the through-gap flow forces the vortex patch close to an edge of the island where it splits into two with only part of the vortex passing through the gap. In general the part squeezing through a narrow gap resymmetrises to have a diameter significantly larger than the gap width.

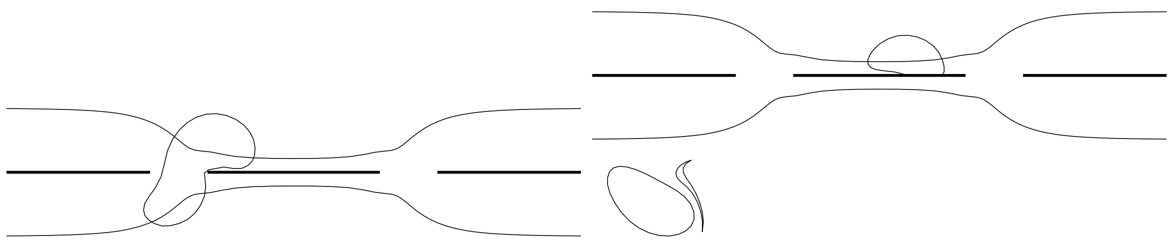

Fig 4. The round-island volume flux forces the larger part of a patch to pass through the first gap even though the patch is 50% wider than the gap.

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MODIFIED SHALLOW WATER EQUATIONS. SIMPLE WAVES AND RIEMANN PROBLEM

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The water flow over the surface is a ubiquitous natural phenomenon such as ocean tides, wind waves, tsunamis, river flows, dam breaks. The usual approximation of common use leads to the shallow water equations. These equations can be nonetheless rarely studied with analytical methods in spite of simplified form the complete hydrodynamic equations. Shallow water equations describing incompressible heavy fluid flow with a free surface on flat plate coincide with those of polytropic gas with the specific heat ratio equals to two. This allows transforming to the classical shallow water theory all continuous solutions for ideal gas equations. This analogy, however, do not apply to discontinuous shallow water flows because of the difference in the conditions on discontinuity surface for this problem. Shallow water flows on plane surface in the presence of the weak vertical inhomogeneities in the initial conditions are studied in present work. Such flows are important for environmental applications. Shallow water approximation for the complete set of hydrodynamic equations in this case contains additional terms appearing as the result of depth averaging of the nonlinear terms in the initial fluid equations and normally has been relaxed in traditional derivation. This new terms describe an advective transport of impulse as a result of the dependence of horizontal shallow water flows on vertical coordinate Averaged nonlinear terms demand parameterization determined by task formulation. The simplest approximation of advective terms of modified shallow water equations allowing full theoretical analysis is suggested in this paper and thus one more alternative way to approximate full hydrodynamic equations in situations when classical shallow-water models fail is suggested. This is especially true when model physics is included in flow equations; in our concrete case suggested parametrisation is applicable in appearance of telegraph type random perturbations of initial conditions.

Traditional derivation of these equations implies hydrostatic approximation for pressure; namely, the vertical pressure gradient balances the gravitational acceleration:

$$p = g \int_0^h \rho dz + p_a.$$

At that rate, we neglect vertical accelerations in comparison with the vertical pressure gradient arising owing to fluid flow. The final step in justifying the shallow water approximation includes depth averaging of transport the horizontal impulse equation and continuity one, and neglecting the nonlinear term:

$$R = \int_0^h \hat{(}u_p - \bar{u})^2 dz = \int_0^h \hat{(}u')^2 dz,$$

that describe the effect of the advective impulse transport caused by difference $u' = u_p - \bar{u}$ of horizontal velocity field u_p from a magnitude of the a depth averaged variable in the shallow water equations $\bar{u}$. As usual this distinction is neglected in the shallow water approximation assuming that horizontal velocities do not depend on vertical coordinate at the initial time and the fluid system is not subjected external disturbances which make it necessary to take into account such terms. Otherwise these terms need to be taken into account by the appropriate parameterization, or it is necessary to solve full system of Euler equations for incompressible fluid.

In the present work we are interested in the influence of the impulse advection as a result of vertical non-homogeneity of initial data, believing initial velocity $u_p(x,z,t)|_{t=0} = u_0(x,z)$ slowly dependent on vertical coordinate and equal:

$$u_0(x,z) = \bar{u}_0(x) + k_0 h_0(x),$$

where $\bar{u}_0(x)$ - initial flow velocity averaged on the depth. In this case the most natural parameterization of advective term in the shallow water equations is $R = k_0 h(x,t)$ and we will introduce the following system of the modified shallow water equations:

$$\begin{cases} \dfrac{\partial h}{\partial t} + u\dfrac{\partial h}{\partial x} + h\dfrac{\partial u}{\partial x} = 0, \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + g\dfrac{(h+H)}{h}\dfrac{\partial h}{\partial x} = 0, \end{cases}$$

here $h(x,t)$ is depth of the fluid layer, u(x,t) is depth averaged fluid velocity , g is gravitational acceleration, and $H = k_0/g$ is reduced factor, characterizing advective transport of the impulse.

In the first part of this work all partial solutions to the modified shallow water equations are found and compared with those of classical one. New effects influencing essentially the limits of applicability of full gas dynamics analogy of classical shallow water equations are found. The special attention is given to the self-similar solutions playing a key role in methods of numerical calculation, taken from gas-dynamics. Solutions obtained allow to formulate and to solve classical initial discontinuity decay problem for this system. In the second part of the present work solutions of Cauchy problem for shallow water equations with piecewise constant initial conditions are found. It is shown that consideration of vertical non-homogeneity excludes one of the solutions configurations that are essential for classical shallow water equations. In particular, in our case there is no vacuum zone characteristic for classical case. We have brought in correspondence with a unique configuration for arbitrary set of the initial conditions $u_1,\ h_1,\ u_2,\ h_2$, i.e. we have determinate flow at every point (x,t).

It is shown, that the initial conditions evolve in two rarefaction waves when

$$u_1 < \varphi(c_2) - \varphi(c_1)$$

holds. The initial conditions evolve in two hydrodynamics when

$$u_1 > \Big(h_1 - h_2\Big)\sqrt{\frac{g}{2h_1h_2}(h_1 + h_2 + 2H)}$$

holds; and when

$$u_1 < \Big(h_1 - h_2\Big)\sqrt{\frac{g}{2h_1h_2}\Big(h_1 + h_2 + 2H\Big)}$$

- in rarefaction wave and hydrodynamics jump. Also the found expressions allow tracing dynamics of the solutions at continuous change of the initial conditions. We discuss other modifications of classical shallow water equations both for theoretical studies and for relevant applications.