DYNAMICS OF SKEW PRODUCTS OF SYMPLECTIC MAPS

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Dynamics of random compositions of symplectic maps $f_k : M \to M$, $k \in K$, can be represented by a single skew product map F of $K^{\mathbf{Z}} \times M$. We study this dynamics when each map f_k is close to integrable. It turns out that understanding chaotic dynamics of F is simpler than of an individual symplectic map f_k.

Possible applications include investigation of almost collision orbits of the n-body problem.

FRACTAL ADVECTION OF PASSIVE SCALARS IN A WAVY JET

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We study mixing, transport and fractal properties of passive scalars in a two-dimensional jet-like flow of an ideal fluid. The model is motivated by Lagrangian mixing and transport in meandering western boundary currents in the ocean like the Gulf Stream in Atlantica and the Kuroshio in the Pacific Ocean. The respective streamfunction

$$\Psi = -\tanh\left[(y - A\cos x)/L\sqrt{1 + A^2\sin^2 x}\right] + C\,y,$$

corresponds to a Bickley jet with a running wave whose amplitude is changed periodically $A(\tau) = A_0 + \varepsilon\cos(\omega\tau + \varphi)$. Where A, L and C are the normalized amplitude, width of a jet and the phase velocity, respectively. The streamfunction, written in the frame moving with the phase velocity, generates the advection equations $\dot{x} = -\partial\Psi/\partial y$, $\dot{y} = \partial\Psi/\partial x$.

We analyze these equations and find all the topologically different regimes of flow and all the possible bifurcations. The geometry and topology of mixing are examined in the range of values of the phase velocity C where the phase portrait consists of two chains of vortices with an eastern wavy jet between them. Taking into account the translational symmetry, it is enough to consider mixing of particles (distributed initially along the straight line with $x = 0$) in the frames with $0 \le x \le 2\pi$ and $-2\pi \le x \le 0$ as a scattering problem. It is shown that both the time T, particles spend in the frames before reaching the lines $x = 0, \pm 2\pi$, and the number of times n they wind around the elliptic points of these zones, have hierarchical fractal structure. Scattering functions are singular on a Cantor set of initial conditions, resembles ones we have found in a scattering problem with a fixed point vortex and an alternating background current [1, 2] but they are much more complicated. We demonstrate in Fig. 1 the fractal $n(y_0)$ and the scattering function $T(y_0)$. The upper parts of the plots of the upper fragment correspond to the particles, entering initially in the eastern frame with, and the lower parts — to the particles in the western frame with. The lower panel demonstrates a zoom of the first levels of the fractal.

In each the frames we recognize a complicated heteroclinic structure with infinity of heteroclinic trajectories and a countable infinite set of unstable periodic trajectories. Stable and unstable manifolds of all these unstable trajectories determine chaotic scattering of passive particles. Points at the ends of all the segments correspond to particles staying in the eastern frame forever. Points close to stable manifolds spend in the frame a long time before reaching the boundaries of the frame. All the particles quit the frame along the unstable manifolds. The geometry of mixing is analyzed with the help of the fractal. Fig. 2 shows the snapshots of the evolution of a material line with the letters corresponding to segments of the fractal in Fig. 1 with $n = 1, 2, 3$. The mixing is the same in all the frames. A correspondence between the dynamical, statistical and topological measures of the flow is established. In conclusion we discuss the role of chaotic mixing and transport in oceanic meandering jet western currents like the Gulf Stream and the Kuroshio.

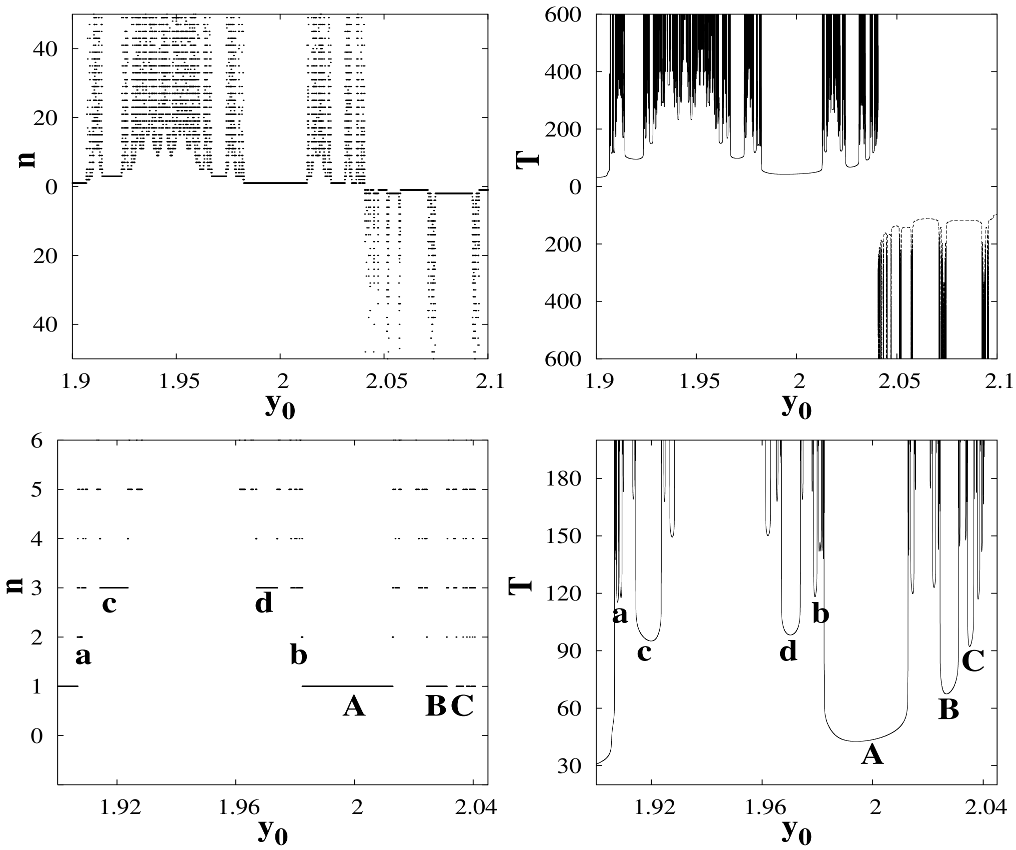

Fig 1. Fractal set of the initial positions y_0 of particles that reach the lines $x = 0, \pm 2\pi$ after n turns around the elliptic points and the respective time of exit T.

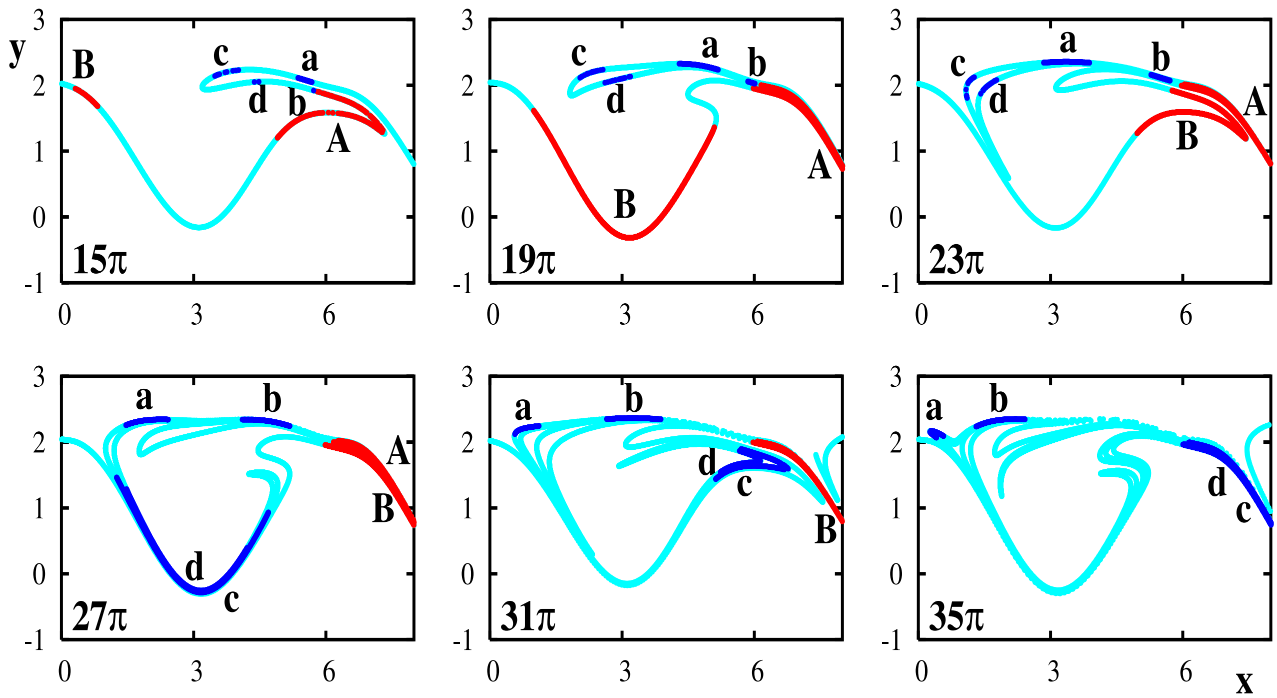

Fig 2. Snapshots of evolution of a material line corresponding to segments of the fractal in Fig. 1 with $n = 1, 2, 3$. The moments of time are shown in plots.

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ROSSBY SOLITARY WAVES IN THE PRESENCE OF A CRITICAL LAYER

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Abstract

Rossby waves are common features of geophysical flows and can be observed in global weather maps in the mid-latitudes of both hemispheres. They make important contributions to nonlinear geophysical dynamics in various ways. Synoptic eddies in the oceans can be modeled by the so-called Rossby solitons, or as barotropic solitary eddies, or as modons, that exist due to a balance between nonlinearity and dispersion due to the Earth's rotation meridional gradient (β-effect) (Kizner, 1997). The long-lived nature of these nonlinear waves depends on the persistence of zonal shearing motions, and is clearly linked to the large-scale coherent features of geophysical flows. This analytical study therefore considers the evolution of weakly nonlinear long Rossby waves in a horizontally sheared zonal current. We examine a stable flow so that the nonlinear time scale is long. These assumptions will enable the flow to organize itself into a long-lived and large-scale coherent structure.

We consider the superposition of a small-amplitude linear Rossby wave on a mean shear flow for which the wave speed equals the mean-flow velocity at a certain latitude in the β-plane. A critical-layer singularity then occurs in the linearized stability analysis, and we examine the subsequent modifications of the flow. The traditional assumption of a weak amplitude breaks down when the wave speed equals the mean flow velocity at a certain latitude, due to the appearance of a singularity in the leading order equation, which strongly modifies the flow in a critical layer. The dynamical processes involved within the critical layer can be expected to play an important rôle in the large-scale dynamics of the atmosphere and the oceans. Rossby-wave breaking generates highly inhomogeneous flows, that is, narrow zones, oriented east-west, which are essentially nonlinear whereas one can observe a wave-like motion outside (Bradshaw et al. 2002). Breaking is also characterized by a rearrangement of the potential vorticity (PV) contours in a more or less irreversible way (McIntyre and Palmer, 1985). Nonlinear critical layer theory is attractive for the modeling of Rossby wave breaking because it implies an analysis of two such coupled regions: an inner flow where nonlinear dynamics plays an outstanding rôle and an outer flow where linear motions prevail. Here, we will focus on outer flows which lead to solitary waves. Matching the inner and outer flows leads to the distortion of the PV contours within the critical layer, and yields the characteristic cat's eyes motions: bounded flows where the streamlines are closed. The symmetry of the critical layer vis-à-vis the critical level is also broken. Previous works: Benney (1966) and Redekopp (1977) showed that the amplitude of long Rossby waves in a shear flow obeyed a Korteweg-de Vries (KdV) equation. Here we expect to obtain a KdV equation, but altered by new nonlinear terms. The evolution of a solitary wave may be complicated in this case. However, in many previous studies, solitary waves have been considered in systems whose dynamics is

modeled by a modified integrable equation: scattering of a shallow water KdV soliton by a depth inhomogeneity, an ion acoustic soliton in an inhomogeneous plasma and acceleration of Langmuir soliton under the action of nonlinear Landau damping (Kivshar and Malomed, 1989).

Redekopp (1977) studied both regular and singular neutral modes but he omitted to the coupling through the streamwise-velocity jump between the critical layer and the outer flow and stopped his expansion at the leading order. As a result, only the usual KdV equation emerged.

We study here the evolution of a free nonlinear singular neutral mode for long space and time scales, both outside and inside the critical layer. Singular neutral modes can be found in the stable linear régime: they are subcritical modes. The β-parameter must be large enough to stabilize the flow. Reynolds stress arguments rule out the existence of singular neutral modes in the linear theory. Nevertheless, singular solutions of the Rayleigh equation exist with a nonlinear critical layer.

An inner asymptotic expansion with scalings valid in the critical layer for a long-time régime is considered in conjunction with the outer expansion. By the method of matched asymptotic expansions, these are matched at the edges of the critical layer. In order to obtain a balance between the effects of quadratic nonlinearity and dispersion, we introduce the scaled variables,

$$X = \epsilon^{\frac{1}{4}} x' , \qquad T = \epsilon^{\frac{5}{4}} t . \tag{1}$$

In this study the singularity is removed in the critical layer by reintroducing advection terms. The equations of motion are analytically integrated at each order of an asymptotic expansion. Integration constants are determined by using an averaging technique on the viscous components which is a generalization by Redekopp (1977) to a solitary wave of the work of Benney and Bergeron (1969). An additional term $\mathcal{C}_X[A]$ appears in the KdV equation for the modal amplitude A, which then has the form,

$$\partial_T A + R_0 A \partial_X A + S_0 \partial_X^3 A + V_0 \partial_X A = \mathcal{C}_X[A] . \tag{2}$$

The extra term $\mathcal{C}$ is a smooth functional of A, and is made necessary at a certain order of the asymptotic expansion by matching the inner flow on the dividing streamlines.

The Reynolds number R is assumed to be very large; indeed, the critical layer scaling will impose $1/R = \lambda\, \epsilon^{\frac{7}{4}}$ where λ is an order-one constant. The continuity of the leading-order vorticity inside the critical layer without the assumption of thin viscous boundary layers all along the separatrices is made possible thanks to small $O(\epsilon^{\frac{1}{2}})$ jumps in the mean vorticity (Haberman, 1972). The existence of such vorticity discontinuity was proved by Brown and Stewartson (1978). We focus on the long-time asymptotic régime after the critical-layer formation stage characterized by the $O(\epsilon^{\frac{1}{2}})$ vorticity spreading throughout the domain by diffusive action. This outward diffusion from the critical layer generates a distorted mean flow. Then transience, nonlinearity or viscosity must be reintroduced in the critical-layer leading-order solution. Here, nonlinearity, together with slow transience, is chosen as being more appropriate for the high Reynolds number flows of geophysical motions. Nevertheless, viscosity is later introduced to render the inviscid solution unique, but the inviscid limit is eventually taken. Within the region of closed streamlines, we invoke a modified form of the Prandtl-Batchelor theorem to determine the interior potential vorticity. The latter is smooth but is no more a constant within the separatrices.

The hypothesis of a slowly evolving motion within the critical layer leads to the derivation of a modified Korteweg-de-Vries equation which is integrable. Solitary wave solutions exist depending on the critical-layer shape, this leads to depression or elevation waves.. However, the matching of the outer and inner flows brings conditions which strongly alter the streamlines geometry in such a way that each streamline is defined by a strained coordinate. This parametrization still reveals to be insufficient to describe the critical layer flow around the dividing streamlines in the case of the elevation wave. A second layer with its own scaling and variable must be then introduced to cancel the existing singularity. This additional layer is not specific to this problem but appears when one wishes to accurately model the motion in the neighbourhood of the separatrices without using viscosity, only by taking into account of nonlinearity. Distortions vanish there contraryly to the depression wave where vorticity and velocity become discontinuous on the separatrices at the orders ϵ^2 and $\epsilon^{\frac{5}{2}}$ respectively.

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CYCLONE INTERACTIONS IN A STRATIFIED ROTATING FLUID

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The instability and interaction of two initially barotropic cyclones is investigated experimentally in a rotating stratified fluid for a large variation in Reynolds numbers between 2000 and 20000 (based on circulation). The vortices under consideration have a close to Gaussian vorticity distribution. Four different type of interactions are represented in the parameter space set by the initial separation distance, d/R, and the Rossby radius of deformation, $R_d = NH/fR$ (where N is the frequency stratification, f the Coriolis parameter and H the height and vortex radius, R). The vortex instability, which depends on deformation radius, varies the critical merger distance and is in good agreement with simulations of [2]. No particular effect of the Reynolds number on the merging has been observed.

A still open question in geophysical turbulence is to which extend meso-scale geophysical flows behave as quasi-two-dimensional flows and tend to a quasi equilibrium state, represented by a horizontally merged and vertically aligned vortices [3, see]. Instability mechanisms, typical for stratified and rotating fluids, may break up larger vortices into small scale structures, thus opposing the organization into large coherent structures, characteristic for the inverse energy cascade. These instabilities were first shown in numerical studies by [2] and [1] and were also

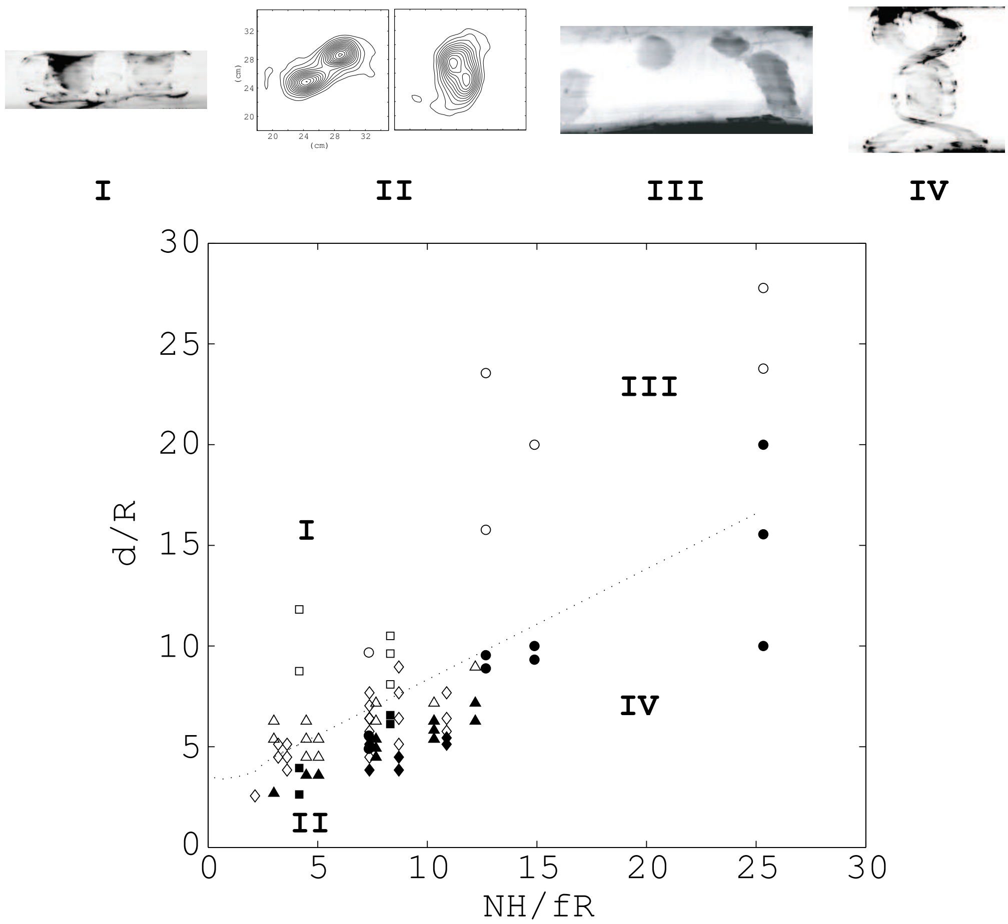

Fig 1. Regime diagram of the cyclone interaction with the initial vortex distance d/R as a function of Burger number $NH/(fR)$. Here, the radius R is based on the second-order-vorticity moment [4]. Filled symbols correspond to merging cyclones. The circles and squares represent experiments in the large tank; triangles and diamonds vortices produced with the cylinders in the small tank. I-IV illustrate the four different regimes, with II the vorticity distribution during merging at mid-depth.

apparent in experimental studies on rake-generated turbulence in rotating stratified fluids [6, see]. In this light we consider individual vortex interactions and their related instability in a rotating stratified fluid.

For these experiments we made use of a small tank of $1 \times 1m^2$ and, to expand to larger Reynolds numbers, the $13m$ diameter rotating platform of the Coriolis team of the LEGI in Grenoble. The cyclones in the small device were generated by removing two volumes (cylinders) from the fluid initially at rest. The removal of the cylinder acts as a local sink, and by conservation of angular momentum results in the formation of a cyclonic vortex for each cylinder. Different cylinder diameters (of 5 and 10cm) resulted in slightly different size vortices and different circulations. The surface was covered with a lid to avoid wind effects induced at relatively high rotation speeds. In the large device the vortices were produced by two rotating flaps [5, see] of about 1m length. Two avoid secondary vortices the flaps were provided with spoilers at their extremity, making the leading edge parallel to the motion. Both vortex generation methods resulted in a cyclonic vortex with approximately Gaussian vorticity distribution. Measurements were made either by recording particle motions in a horizontal plane at mid-depth and processed with the PIV method to obtain the velocity field, or using dye visualizations (see figure 1).

The main result is the diagram presented in figure 1, showing a critical merging distance that is close to the result [1] obtained for ellipsoids of constant potential vorticity vortices in a quasi-geostrophic flow (dashed line). The four different type of interactions (I-IV) represent: I no merging, II merging over the entire fluid depth, III breaking of the vortex structure into patches and domes attached to the free surface and bottom, and IV local merging due to vortex instability. In case I the vortices do not merge, even on relatively long time scales. The side-view dye-visualizations in figure 1 (III, IV) show that the flow becomes three-dimensional due to the tall column instability [2, see] which develops for normalized radii of deformation (aspect ratio) higher than 3. Note that the merged vortex has a lower aspect ratio (IV). The interactions for I and II are in good approximation two-dimensional. This instability and its influence on the merger is almost independent of the Reynolds number and the shape of the vorticity distribution as far as it is cyclonic. Also the fact that the Rossby number is of O(1) has no significant influence on the developpment of the instability. Additional observations concern filamentation and inertia gravity wave radiation.

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VORTEX INTERACTION IN AN UNSTEADY LARGE-SCALE SHEAR/STRAIN FLOW

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Vortex interactions in geophysical turbulence and in the ocean can be very nonlinear and lead to the disappearance of one or several of the initial vortices. In vortex merger for instance, the end-product of the horizontal interaction of the two initial vortices can be one larger vortex surrounded by filaments. Vortex merger has been identified as the process by which energy cascades towards larger scales and enstrophy towards smaller scales in 2D incompressible turbulence (McWilliams, 1984). Therefore, it has received considerable attention.

Theoretical and numerical work has shown that two Rankine vortices will merge if they are initially closer than 3.2 radii (Dritschel, 1985 ; Melander et al., 1988). Spread-out vortices (such as Gaussian vortices) can merge at larger distances due to their vorticity tail (Caperan and Verron, 1988). This first view of vortex merger was manichean : vortices could only co-rotate or merge. Further work has shown that several mechanisms can be antagonistic to vortex merger, such as the influence of bottom topography or a curved free surface, of stratification or of beta-effect (Carnevale et al., 1991 ; Bertrand and Carton, 1993 ; Verron and Valcke, 1994). Such influences would lead to a richer outcome of the interaction, including partial vortex merger, dipole formation... In turbulence and in the ocean, vortex pairs are rarely isolated. The effect of surrounding vortices on a given pair is comparable to a large-scale shear or strain field. Adding such a field to the problem of two-vortex interaction leads to the appearance of stationary positions for equivalent point vortices (and also for some specific vortex shapes). Vortex merger in a large-scale steady shear/strain field has recently been investigated (Maze et al. 2004).

The present work will come closer to reality by considering that the surrounding vortices are not fixed and therefore exert a time-varying shear and strain field on the vortex pair. A first step considers the motion of two point vortices (as indicative of the motion of finite- area vortex centers) in a periodic large-scale field. It is shown that harmonic or subharmonic forcing can lead the vortex pair to migrate from the vicinity of the fixed points to the vicinity of the center of the plane. The regularity (or chaotic nature) of such motions is investigated. As a second step, a numerical model of finite-area vortices is used to quantify the influence of the time-varying large-scale field on the domains of existence of the various nonlinear regimes (co-rotation, merger, ejection, trapping near a fixed point), and also on their rate of erosion via filamentation. Finally, the influence of a stochastic large-scale field is more briefly addressed.

TOPOLOGICAL ASPECTS OF THE VORTEX STRUCTURE OF TORNADO

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Summary

It examine the topological aspects of a concentrated vortex structure, a tornado, in inviscid fluid. A laboratory study is complemented by numerical simulations of vortex dynamics taking into account the topological structures of vortex knots. Natural observation performed in Kazakhstan (August 2002) agree reasonably well with laboratory experiments.

Tornado vortex structure

It have considered three-dimensional concentrated vortex problem related to the topological properties of intensive tornado structures.

Fig 1. Tornado in Kazakhstan (August 2002)

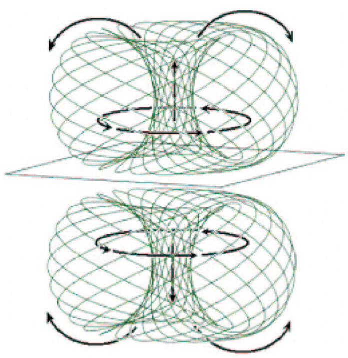

Fig 2. Model of vortex structure

It is assumed that initially the tornado can be modelled as a torus (two-dimensional closed manifold) over a rigid surface. The mirror image of the torus (Figure 1) provides the possibility to consider the development of this vortex structure in an unbounded domain. It have performed the experimental studies of the tornado-like structures using the suggested idea. The phenomenon of vortex hysteresis is experimentally observed. The numerical model [2] may be readily extended to describe more complex physical situation. The Lyapunov stability conditions is essential while studying the Euler characteristic of the vorticity field. The possibility of effective stirring by a steady motion in the vortex core is pointed out. These problems might be benchmark ones for various numerical schemes in the vortex dynamics.

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NONLINEAR STABILITY OF META-STABLE SHOCK WAVES

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Constructing of solutions of standard self-similar problems of the nonlinear theory of elasticity revealed a none-uniqueness of these solutions [1, 2]. For some range of parameters the existence of two solutions was found. One of these solutions is a meta-stable shock wave, and, in accordance with the conservation laws, it can decay (disintegration) into a system of waves propagating with different velocities. In the domain of the none-uniqueness of solutions, this system of waves is the second of solutions we found. A possibility of the shock wave decay doesn't mean that the shock wave should decay necessarily. Such situations have been found in the studies of the wave processes in various problems of the continuous mechanics theory, and it was shown that under certain conditions, the meta-stable states can exist for a long time interval. To answer a question of possibility of existence of the meta-stable shock waves and for investigation of the uniqueness of solutions of the corresponding physical problems, we need more detail analysis of these problems with various small scale phenomena which smooth the discontinuities to be taken into account. In this paper, we use for these purposes the viscosity described by some dissipative terms inserted into the nonlinear hyperbolic equations under consideration. In this case, the nonlinear-elastic media is considered as a limit of a viscose-elastic media with vanishing viscosity. A set of numerical experiments were made for analysis of stability of a viscose structure of meta-stable shock waves. The numerical experiments showed a stability of these shock waves with respect to one- and two-dimensional perturbations of small as well as of finite amplitudes.

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CHAOTIC ADVECTION AND NONLINEAR RESONANCES IN A PERIODIC FLOW ABOVE A SUBMERGED OBSTACLE

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In the framework of background currents on an f-plane [1], considered is given to a dynamically-compatible model of a periodic flow over an isolated submerged obstacle of Gaussian shape. The stream function may be written in the following form

$$\Psi = W_0\Big[1+\varepsilon\sin\big(\omega t+\varphi\big)\Big]\Big[\big(W_1\sin\theta+u_0\big)x-\big(W_2\cos\theta+v_0\big)y\Big]+\int\limits_0^r V(\rho)d\rho, \qquad (1)$$

where $\theta=\theta(t)$ is a given function of time (for example, $\theta=\theta_0 t$), W_0, W_1, W_2, ω, φ, θ_0, u_0, v_0 are constants, $r=\sqrt{x^2+y^2}$, x and y are horizontal coordinates directed to the east and north, respectively, $V(r)$ is a radial distribution of azimuthal velocity in a vortex induced by the obstacle:

$$V=\frac{\sigma}{2\alpha r}\Big(e^{-\alpha r^2}-1\Big).$$

Here σ, $\alpha=O(1)$ are constants.

For the purposes of the calculation, it is assumed that $V'(1)=0$ (where from it follows $\alpha\approx 1.256$) and that the current over the obstacle performs diurnal oscillations. If we take typical values for the average depth of the ocean and the azimuthal velocity to be 4 km and 10 cm/s respectively, we obtain $\sigma=3.511$, a value corresponding to an obstacle height of 1.021 km. The velocity vector of the model tidal current (1) performs an ellipse. In the undisturbed case ($W_1=W_2$, $\varepsilon=u_0==v_0=0$) the center of the rotation coincides with the center of the submerged obstacle, and the current occurs to be circular. Conditions may be derived which imply the possibility of topographic vortices being generated over the obstacle. It is shown that in a certain interval of velocities W_0 of a circular external current there can exist two vortices with oppositely-directed circulations, which move around the center of rotation along a circular trajectory. When the ratio of the frequency of a fluid particle rotation in the vortex to the frequency of the external current (θ_0/ω) is rational, the trajectories of fluid particles are closed and periodical, providing a non-finite number of quasi-stationary solutions. Dynamic chaos generation in the system has been studied. It has been shown that either changes of the external current velocity ($\varepsilon\neq 0$) or the elliptic character of the tidal current ($W_1\neq W_2$) or the displacement of the rotation center from the obstacle center (u_0, $v_0\neq 0$) may play the role of a non-stationary disturbance and correspondingly lead to the chaotization of a number of trajectories. Moreover, the study reveals the appearance of nonlinear resonances, also caused by the non-stationary system disturbances, which occur in the vicinity of periodic stationary trajectories. The non-linear resonances are realized as vortices-satellites having the same multiplicity (number of vortices rotating around a common center or around a topographic vortex) as corresponding stationary trajectories. The stability of coherent structures (topographic vortices, nonlinear resonances) to different types of disturbances has been analyzed.

As an example, the figure 1 shows the co-rotating Poincaré section for a current with $\varepsilon = u_0 = v_0 = 0$, $W_0 = 0.92$, $W_2 = 1.7$, $W_1 = 1/W_2$, $\theta_0 = 0.14$. A circle in the figure of unit radius defines the external boundary of an effective submerged obstacle.

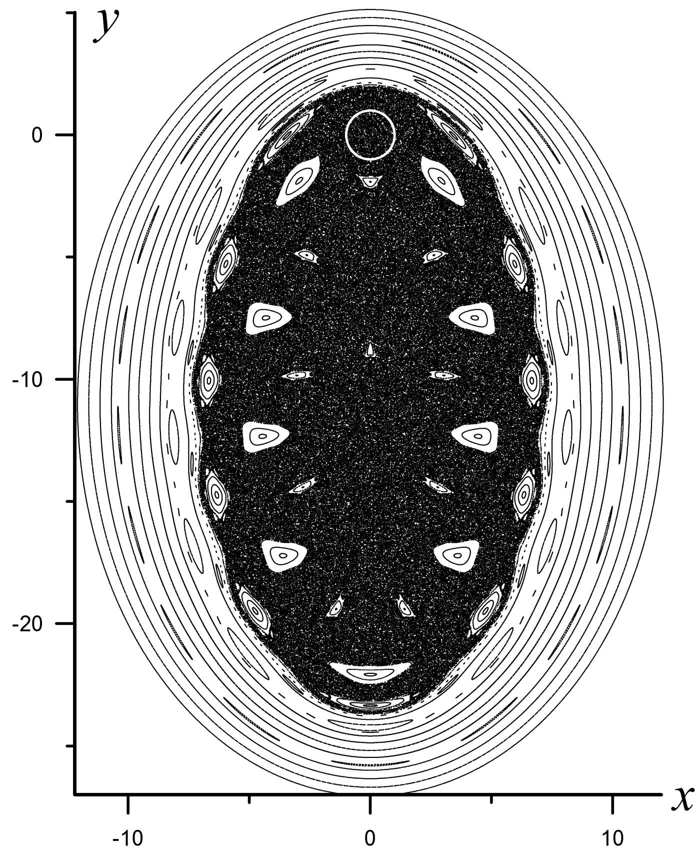

Fig 1

This investigation is the extension of [2], where we essentially studied regular peculiarities occurring in a periodic current over an obstacle.

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