# NONINTEGRABILITY AND FRACTIONAL KINETICS ALONG FILAMENTED SURFACES

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Invariant surface, with respect to Hamiltonian dynamics, is wound by a particle trajectory in phase space. The surface is filamented if it has a topological genus more than one, i.e. if it is equivalent to a sphere with more than one handles. Dynamics of particles or field lines along the filamented surfaces can be found in fluids, tokamak plasmas, and solar corona. Geodesics along the filamented surfaces are not integrable [1]. Similar statement can be conjectured for trajectories that wind filamented surfaces. In some simplified situations the nonintegrability of the dynamics corresponds to a kind of randomness with zero Lyapunov exponent, called pseudochaos. We consider a situation when the study of pseudochaotic dynamics can be reduced to study dynamics in rectangular billiards with bars (slits) inside. All results are formulated for ensemble of trajectories with irrational tangents. Trajectories in these billiards can be considered applying Diophantine approximation. It is speculated, on the basis of simulations, that trajectories are "sticky" to some periodic trajectories, i.e. real trajectories have long almost periodic parts with periods that appeared from their Diophantine approximants.

Our results are obtained for the probability density $P(t)$ of the Poincare recurrences for ensemble of irrational trajectories that are wandering along the filamented surfaces. The large time $t\to\infty$ asymptotic of the probability density $P(t)$ is a power-wise function: $P(t) = \text{const}/t^{\gamma}$, and the power $\gamma$ i.e. recurrence exponent, depends on the number of filaments $M$. The method to calculate the exponent $\gamma$ is based on the Diophantine approximation and the renormalization group equation (RGE) obtained for the distribution of the Poincare recurrences. The expression for $\gamma(M)$ is obtained for large $M$. On the basis of these results we were able to propose a fractional kinetic equation (FKE) and estimate the so-called transport exponent, i.e. the exponent that defines the mean displacement dependence of particles during large time. The FKE is an analog of the diffusion equation but, in contrary to it, the derivatives with respect to time and displacement are generally speaking not integer. The technical part of the paper is based on the generalization of the RGE for $P(t)$ and the scaling properties of continues fractions. The results are continuation of works [2, 3], and they are compared to the simulations performed in the same works.

The phenomenon of stickiness of trajectories is discussed from a general concept of dynamics in the presence of singular zones in the phase space while the singular zones appearance is due to divergence of moments of $P(t)$ beginning from $m \geq \gamma + 1$.

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# PART II. SECTIONAL TALKS

## WEAKLY TURBULENT LAW OF WIND WAVE GROWTH

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Dominating role of the nonlinear transfer for growing wind-driven waves is discussed in a number of theoretical and experimental papers (see e.g. [2, 1]). A key outcome of the dominance is a tendency of wave spectra to keep a self-similar form. This physical fact is built into conventional experimental parameterizations of wind-wave spectra as a typical incomplete self-similarity: the spectral shape is assumed to be a quasi-universal one while the total energy content depends monotonically on a wave age parameter $C_p/U_h$ ($C_p$ is phase speed at peak frequency, $U_h$ is wind speed at some reference height $h$). The dependence on wave age describes the effect of wind forcing and dissipation on spectral growth. Theoretically, the problem can be treated within an asymptotic approach for the kinetic equation for wind-driven waves (the Hasselmann equation). It leads to the splitting of wind-wave balance into two parts:

$$dN_k/dt = S_{nl}; \qquad d\langle N_k\rangle/dt = \langle S_{in} + S_{diss}\rangle \tag{1}$$

Here $N_k$ is wave action spectral density, $d/dt$ is full derivative and angle brackets mean integration over the whole wavevector space. First equation (1) says that spectral shape is determined by nonlinear transfer term $S_{nl}$ only while the second one describes evolution of total wave action under wave input $S_{in}$ and dissipation $S_{diss}$ and plays a role of a specific boundary condition that selects the spectral shape and the growth rate of a particular solution.

Self-similar solutions for (1) can be found when total wave action is a power-law function of time or fetch:

$$N(\mathbf{k}, t) = at^{\alpha}U_{\beta}(b\mathbf{k}t^{\beta}); \qquad N(\mathbf{k}, x) = ax^{\alpha}U_{\beta}(b\mathbf{k}x^{\beta}) \tag{2}$$

In (2) $a$ and $b$ are parameters that should be related to "boundary condition" (second eq.1): characteristics of external forcing. It leads to time- (fetch-) independent weakly turbulent self-similarity law that in terms of total wave energy takes a following form

$$\varepsilon\, \omega_*^4/g^2 = \alpha_{ss}\left[(\omega_*^3\, d\varepsilon/dt)/g^2\right]^{1/3} \tag{3}$$

Eq. 3 relates total energy $\varepsilon$ and characteristic wave frequency $\omega_*$ to total flux $d\varepsilon/dt$. The relationship (3) can be extended to a general case as an adiabatic approximation for the system (1).

The self-similarity parameter $\alpha_{ss}$ depends on parameters of wave growth, but this dependence is rather weak due to the property of quasi-universality of spectral shapes implied by the conventional parameterizations of wind-wave spectra [2] and justified in an extensive numerical study [1]. The asymptotic weakly turbulent law (3) has been verified for numerical results on duration-limited wind-wave development and for available experimental fetch-limited power-law dependences of wave growth collected for more than 40 years [3, 4].

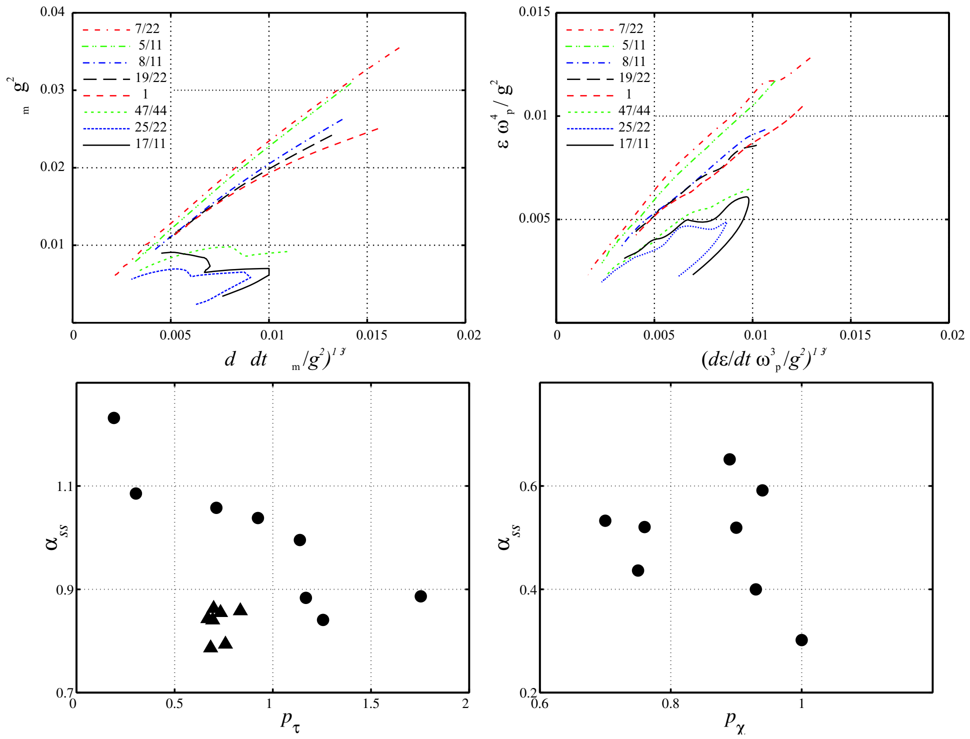


Fig 1. Top raw — non-dimensional energy *vs* non-dimensional wave input in numerical runs with different exponents of total energy temporal growth (in legend), mean (left) and peak (right) frequencies are used for scaling. Bottom raw — estimates of $\alpha_{ss}$ in (3) for numerical results (left, • — artificial, ▲ — "realistic" $S_{in}$) and for fetch-limited field experiments (right).

The trend to the asymptotic law (3) is illustrated by the upper raw of fig. 1 for a series of numerical runs for different exponents of total energy growth (in legend). The peak frequency $\omega_p$ appears to be more representative for scaling (top right) as compared with mean over spectrum frequency $\omega_m$ (left top). Estimates of self-similarity parameter $\alpha_{ss}$ for numerical solutions of homogeneous kinetic equation (left bottom) and for experimental power-law approximations of wave energy and frequency in fetch-limited field experiments (right bottom) gives a reasonably low dispersion of the estimates and rather close values of $\alpha_{ss}$ for the cases of temporal and spatial wave development. This validates general importance of the weakly turbulent law of wave growth (3).

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## SHOCKS WITH REQULAR AND STOCHASTIC STRUCTURES IN NON-DISSIPATIVE AND LOW-DISSIPATIVE DISPERSIVE SYSTEMS

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Nonlinear non-dissipative and low-dissipative systems with dispersion are investigated. The equations under consideration contain high-order derivatives. Typical equation is the generalized Korteweg-de Vries-Burgers equation

$$a_t + b_1 a_x + (a^2/2)_x + b_3 a_{xxx} + b_5 a_{xxxxx} = \epsilon a_{xx}$$

This equation is a model equation for description of various phenomenon. For example it describes propagation of shallow-water waves under an ice cover. The other models such as equations of cold plasma, equations of composite material and generalized nonlinear Shredinger equation were investigated also. Direct numerical simulation for step-like initial data showed that there are solutions that contain shock structures. For dissipative systems the shock structure is transition between two homogeneous states. For non-dissipative systems the shock structure is any transition between homogeneous, periodic, multi-periodic or stochastic states. It is assumed that states under consideration are described by some simplified or averaged equations with first-order derivatives and with slow variables $T = \delta t$, $X = \delta x$, $\delta << 1$. Methods to derive and solve such systems and methods to get shock structure solutions and boundary conditions on shocks are developed. Shocks can be observed in numerical solutions of corresponding partial differential equations then $t \to \infty$ if initial data is step-like (for example $a = \tanh x$). For low-dissipative case the solution under consideration is stationary, statistically stationary or time-periodic then $t \to \infty$. For non-dissipative case the solution can be self-similar or stochastic. The other way is investigation of numerical solutions of ordinary differential equations that describe stationary solutions (travelling wave equations). Note that the first way permits to examine existence and stability of the shock while the second way shows only existence. So investigations of shocks are investigations of attractors and bifurcations of finite and infinite dimensional dynamical systems.

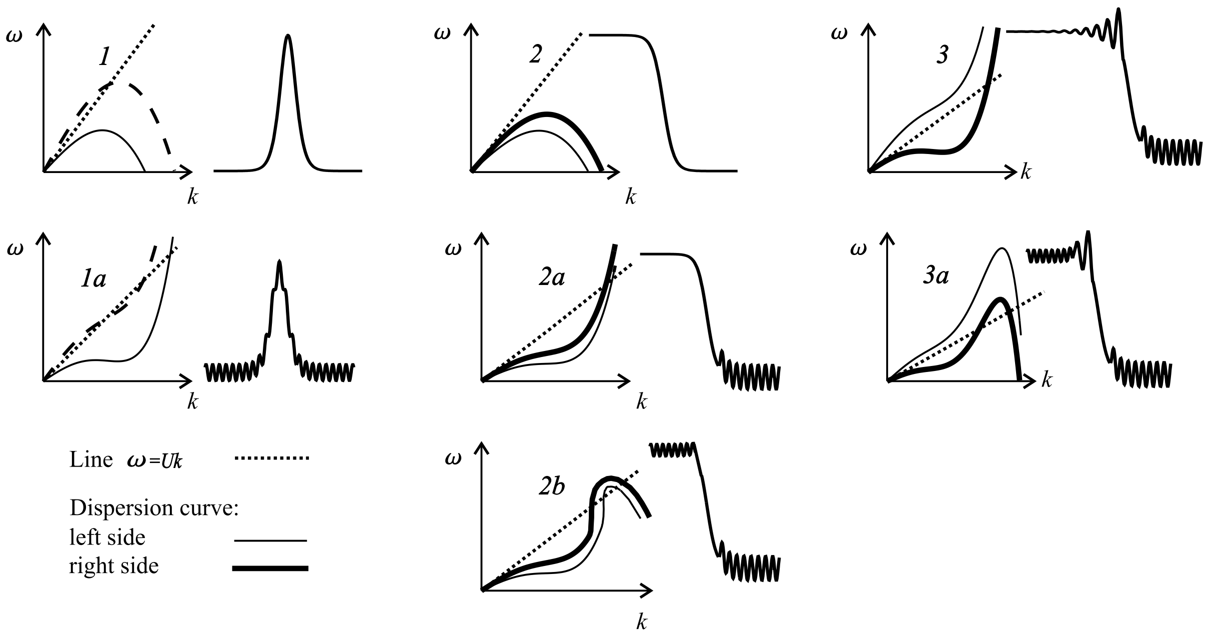


Fig 1

The method to predict shock structure type is developed. It is based on the analysis of dispersion curve of linear equations. There are seven evolutionary (stable) types of regular (non-stochastic) shocks. The type of shock depends on the number of intersections between dispersion curve $\omega = \omega(k)$ and the line that corresponds to phase speed of the shock $U = \omega/k$. The solitary wave type shock (fig. 1, *1*) is a transition between homogeneous state and sequence of solitary waves. There are no intersections for side 1 (right side from the shock) and one intersection for side 2 (left side) in this case. Kink (fig. 1, *2*) is a transition between two homogeneous states, no intersections for side 1 and no intersections for side 2. Shock with radiated wave (fig. 1 ,*3*) is a transition between uniform and periodic state, no intersections for side 1 and one intersection for side 2 . Kink with radiated wave (fig. 1, *2a*) is a transition between uniform and periodic state, one intersection for side 1 and one intersection for side 2. Shock with two radiated waves (fig. 1, *3a*) is a transition between two periodic states, one intersection for side 1 and two intersections for side 2. Kink with two radiated waves (fig. 1, *2b*) is a transition between two periodic states, two intersections for side 1 and side 2. In other cases shock structures are structures with stochastic behaviour caused by multiple resonance interactions of radiated waves are predicted. Example of such a solution for some large value of $t$ is given below (fig. 2). This solution is observed instead of regular solution with so-called generalized solitary wave (fig. 1, *1a*).

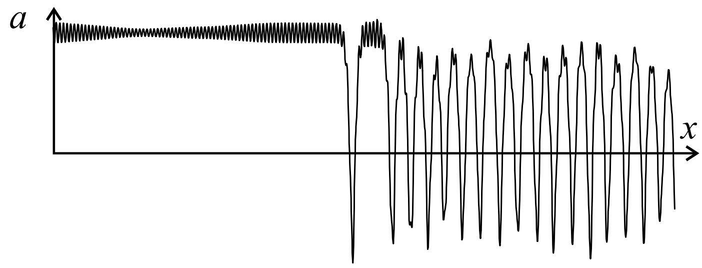


Fig 2

For the case of non-dissipative systems these statements are used directly and for low-dissipative systems they are used for the analysis of internal shocks of dissipative shock structures. For rapid variables the system can be treated as a non-dissipative one so the non-dissipative shock structure can be included as internal structure for construction of dissipative shock structure described by averaged equations with slow variables. Dissipative terms in the low-dissipative case must be withdrawn for the analysis of the dispersion curve. Example of regular solution with the shock with radiated wave is given below (fig. 3).

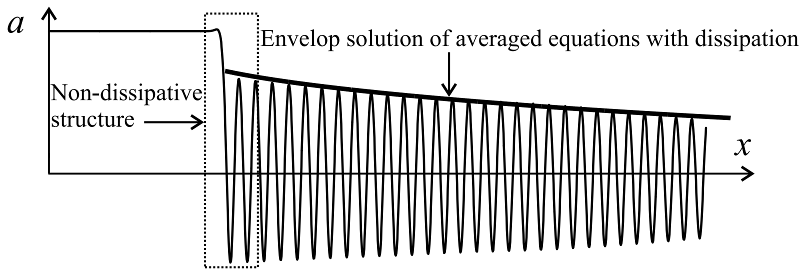


Fig 3

Direct investigation of travelling wave ordinary differential equations shows the existence of shocks with resonance two-wave states. These shocks are not predicted by the method described above but they are observed as internal shocks in low-dissipative shock structures.

For the case of finite dissipation time-periodic shock structure solutions are observed. These solutions are caused by attraction of two different resonance solutions.

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# RECONSTRUCTION OF FLOW TOPOLOGY AND PERCOLATION SCALINGS

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The turbulent diffusion models differ significantly from one-dimensional transport models. Often several different types of transport are present simultaneously in turbulent diffusion. A variety of forms requires not only special description methods, but also an analysis of general mechanisms for different turbulence types. One such mechanism is the percolation transport [1]. Its description is based on the idea of long-range correlations, borrowed from theory of phase transitions and critical phenomena. These long-range correlations are responsible for the anomalous transport. It was suggested that we could explain anomalous transport in two-dimensional cases in terms of the percolation threshold. In the present paper we consider the influence of drift flow and time-dependence effects on the passive scalar behavior in the framework of the percolation approach. The renormalization method of a small parameter is reviewed in continuum percolation models [2, 3, 4]. It is suggested to modify the renormalization condition of the small parameter of the percolation model in accordance with additional external influences superimposed on the system. This approach makes it possible to consider simultaneously both parameters: the characteristic drift velocity $U_d$ and the characteristic perturbation frequency $w$. The effective diffusion coefficient D is proportional to $w^{1/7}$ that satisfactory describes the low-frequency region w, where the long-range correlation effects play a significant role. The character of the dependence of $D_{eff}$ on the drift flow amplitude $U_d$ in different regimes is analyzed [4, 5].

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# LAGRANGIAN FLOW GEOMETRY OF TRIPOLE VORTEX SIMULATIONS

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The tripole is a two-dimensional flow structure consisting of a linear arrangement of three vortices, of alternating sign. The whole structure rotates in the direction of the core vortex rotation. It has been observed in the laboratory in rotating [6, 7] and stratified fluid [3], where it is the product of growth and saturation of the instability of a shielded monopolar vortex. Tripole generation from unstable monopoles has also been addressed in numerical studies [2, 4]. More recently, the tripole vortex was observed in the destabilization of a Gaussian monopole by a strong quadrupolar perturbation [5]. In this case, the structure does not have total circulation equal to zero ("shielded" case), but rather can have satellites of varying strength. The amplitude of the quadrupolar component in the initial condition determines whether the flow will evolve into a monopole or a tripole, and the existence of a critical amplitude has been conjectured [5]. A parameter study with the goal of determining this critical value for different Reynolds numbers has been performed and is being prepared for publication [1]. Here we analyse the Lagrangian flow geometry of the tripole vortex, under varying strengths of the satellite vortices; *cf.* Figure 1(b). By looking at the hyperbolic trajectories and their stable and unstable manifolds, calculated from the numerically generated time evolving velocity field, we make several observations regarding the tripole. When the amplitude of the initial perturbation is large enough, the stable manifolds fold and wrap around the areas of negative vorticity, thereby forming a barrier for their mixing. We also note that the Lagrangian and Eulerian flow geometries differ appreciably, and thus it is not correct to ascribe the permanence of the satellites to the formation of a "critical separatrix", as argued previously [1]. In fact, the separatrices are there at the initial time, even in cases where the flow axisymmetrizes. The steadiness of the flow is assessed using scatter plots of vorticity versus stream function, as shown in Figure 1(a). As a quasi-steady tripole is approached, the Lagrangian and Eulerian geometries are more alike, as in the last frame of Figure 1(c). Similar and additional observations for various cases will be presented in the final paper.

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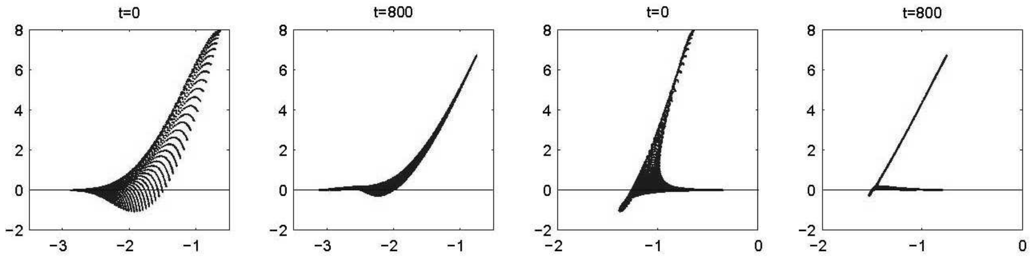


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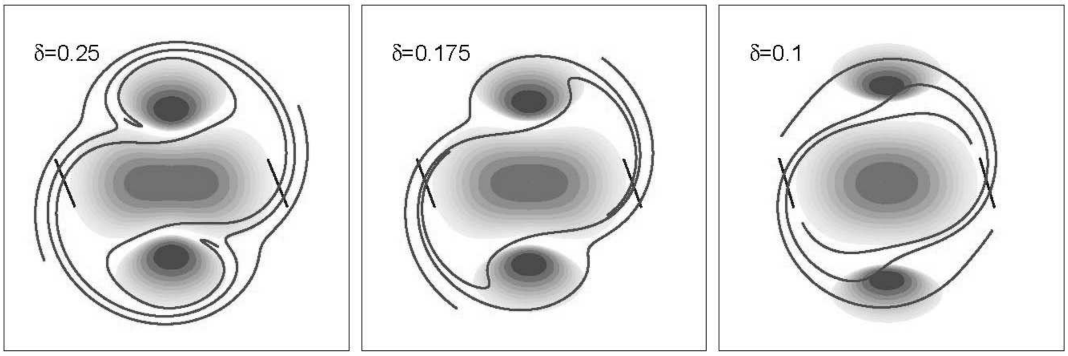


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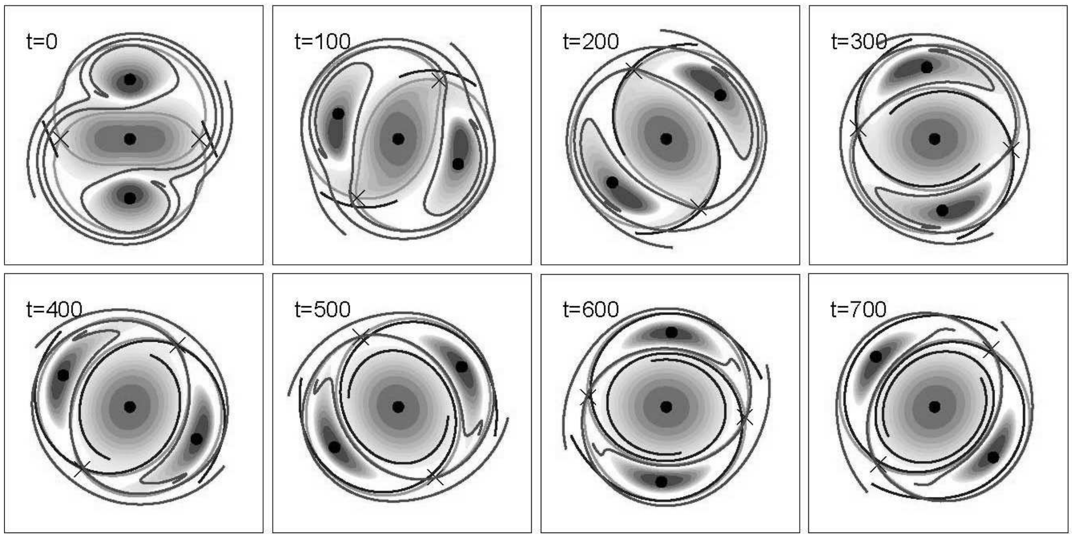


*c*)

Fig 1.
*a*) $(\omega, \psi)$ scatter plots for a tripole with $Re = 3 \times 10^3$, and $\delta = 0.25$ the amplitude of the initial perturbation; two left frames: uncorrected, two right frames: corrected for the tripole rotation. At $t = 800$, the structure is quasi-steady.
*b*) Stable manifolds of the tripole's hyperbolic trajectories for three different amplitudes of the initial perturbation. Only the left-most case develops a quasi-steady tripole.
*c*) Stable and unstable manifolds, and Eulerian geometry (green) for the time-evolving tripole with $Re =$ $= 3 \times 10^3$, and $\delta = 0.25$.

## CASCADES OF PERIOD MULTIPLYING IN PLANAR HILL'S PROBLEM

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The problem of period multiplying cascade detection in some dynamical system is rather complicated. The presence of bifurcation chain is not a guarantee that this chain will continue ad infinitum. The indirect confirmation of infinite period multiplying cascade presence is self duplication of the period multiplying "tree" and convergence it's characteristics to the universal values.

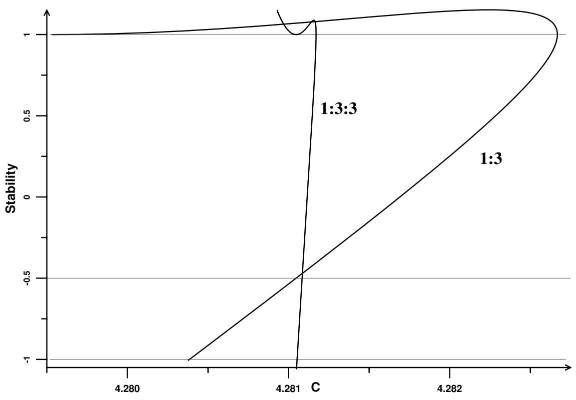


Fig 1. Stability indices versus Jacobi constant for resonances 1 : 3 and 1 : 3 : 3

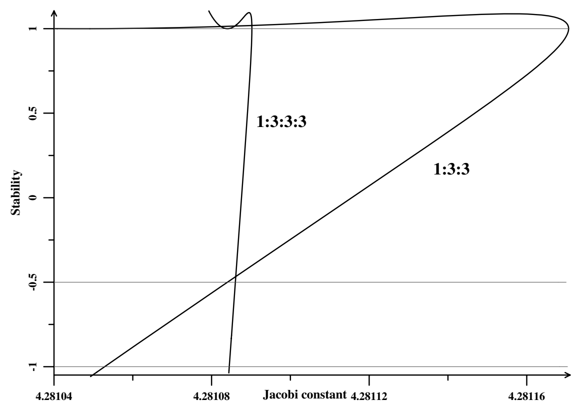


Fig 2. Stability indices versus Jacobi constant for resonances 1 : 3 : 3 and 1 : 3 : 3 : 3

Earlier with the help of methods described in [1] the authors have found out and investigated period doubling bifurcations in the planar circular Hill's problem. This problem is a particular case of the well-known in celestial mechanics restricted three body problem which demonstrates different scenarios of transition from regular to chaotic forms of motion. More over, the invariance of the Hill's problem Hamiltonian

$$H(q_1, q_2, p_1, p_2) = \frac{1}{2}\left(p_1^2 + p_2^2\right) + q_2 p_1 - q_1 p_2 - q_1^2 + \frac{1}{2} q_2^2 - \frac{1}{r}, \text{ where } r = \sqrt{q_1^2 + q_2^2},$$

under canonical transformations

$$(t, q_1, q_2, p_1, p_2) \to (-t, -q_1, q_2, p_1, -p_2)$$
$$(t, q_1, q_2, p_1, p_2) \to (-t, q_1, -q_2, -p_1, p_2)$$

essentially simplify searching for period multiplying bifurcations (see [1]). For instance, the period doubling sequence $1-2-4-\ldots-512$ were built by the authors for the family of periodic orbits $g'$ [2] using the Poincaré section technique [3]. The sequence of the bifurcation values Jacobi constant is quickly converged to its universal limit $\delta \approx 8.721\ldots$ together with other scaling constants $\alpha$ and $\beta$.

The goal of the present work is study of other period multiplying sequences, e. g. period tripling and mixed (tripling of doubling and vice verse) as well as numerical determination their of Feigenbaum and scaling constants. The isoenergetic reduction of the phase flow on the Poincaré secant plain prevents from the successfully continue the branching out periodic solution multiplicity more than 2. That is why the authors apply the method of branching and continuation of second kind periodic solution based on the thorough analysis of generating solution monodromy matrix [4]. The scaling constants are computed using high precision arithmetic because of bifurcating orbit period rapid growth. The pictures below demonstrate the similarity of stability indices behavior for the orbits of period tripling cascade.

The obtained results allow to state that transition to chaotic form of motion in the Hill's problem might happen through the infinite period multiplying cascades.

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## STATISTICAL MECHANICS OF VORTEX LINES

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The major task of turbulence theory is to establish the governing equations for slow varying characteristics of turbulent motion. The current understanding of the dynamics of turbulent flows is still not sufficient to properly perform this task. However, in one special case, motion of ideal incompressible fluid in a closed domain, a certain progress has been made. For two-dimensional motion, the presentation of fluid dynamics as the dynamics of a large number of point vortices along with the ergodic hypothesis yield the equations for averaged fluid flow. The derivation of these equations and their solutions have been intensively investigated (see, e.g., a review in [1]). Much less is known for three-dimensional flows. Here the main results were obtained in [2, 3, 4]. Similarly to the two-dimensional case, the dynamics of fluid is "discretized" by replacing it with the dynamics of a large number of vortex lines. The major motivation for such discretization is that it allows one to automatically satisfy to an infinite set of integrals of fluid motion - conservation of vorticity. Statistical mechanics of fluid motion becomes statistical mechanics of the "particles" with complex structure: each "particle" is a line. Invoking the ergodic hypothesis, one can obtain the equations for averaged characteristics of fluid motion. Three-dimensional flows differ drastically from the two- dimensional ones. A remarkable feature of the three-dimensional flows is that Schrödingers type equation appears as a part of the system of equations for averaged velocity field. This feature has a simple origin: As was shown by R. Feynman, Schrödingers equation appears naturally to describe the result of the summation of some functional of particle trajectories over all possible particle paths. Computing the phase volume of the phase space of vortex lines, one faces the problem of summation over all possible positions of vortex lines for the functional which is similar to that of quantum mechanics. Accordingly, some eigenvalue problem plays a key role in averaged description of three-dimensional fluid motion. The talk will review the results obtained in this area and present the recent ones concerning turbulent motion in pipes and channels.

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## NON-INTEGRABLE PERTURBATIONS OF THE DYNAMICS OF TWO POINT VORTICES

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It is well known that the Hamiltonian dynamics of two point vortices in an ideal fluid in the standard half-plane $\mathbb{H}^2 = \mathbb{H}_0^2 := \{z = x + iy = (x, y) \in \mathbb{R}^2 : y > 0\}$ located at points $z_1$ and $z_2$ and having nonzero vortex strengths $\Gamma_1$ and $\Gamma_2$, respectively , which we represent in the two-degree-of-freedom form

$$\dot{q} = \frac{\partial H_0}{\partial p} = \{H_0, q\}, \quad \dot{p} = -\frac{\partial H_0}{\partial q} = \{H_0, p\}, \tag{1}$$

is completely integrable in the Liouville-Arnold sense, because it has two independent, involutive constants of motion; for example,

$$H_0 \text{ and } J := \Gamma_1 p_1 + \Gamma_2 p_2. \tag{2}$$

It is natural to ask what are the simplest forms of Hamiltonian perturbations to this system that produce chaotic - and therefore non-integrable - dynamics. A perturbed form of the system (1) may be written as

$$\dot{q} = \frac{\partial (H_0 + H_1)}{\partial p} = \{H_0 + H_1, q\}, \quad \dot{p} = -\frac{\partial (H_0 + H_1)}{\partial q} = \{H_0 + H_1, p\}. \tag{3}$$

Conditions on the perturbation $H_1$ are described that guarantee that (3) - unlike (1) - has chaotic solutions. Properties of the perturbation that are sufficient for the existence of chaos - which in general preclude complete Louisville-Arnold integrabilty - are derived using a Melnikov type argument. These results are then applied to the problem of finding nonstandard half-planes of the type

$$\mathbb{H}_\varphi^2 := \left\{(x, y) \in \mathbb{R}^2 : y > \varphi(x) \; \forall x \in \mathbb{R}\right\},$$

where $\varphi : \mathbb{R} \to \mathbb{R}$ is a nonnegative continuous function, on which the two point vortex problem can exhibit chaotic solutions. Several numerical simulations are presented to illustrate the findings.