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The collection of talk abstracts of the IUTAM Symposium “Hamiltonian dynamics. Vortex structures. Turbulence” consists of two parts. Plenary lectures are submitted in the first part, sectional talks are collected in the second one. In both parts the abstracts are placed in the alphabetic order of the last name of the first author.

At the end of the collection, there is given the index of all authors.

PART I. PLENARY LECTURES

VORTEX DYNAMICS OF WAKES

HASSAN AREF

Department of Engineering Science and Mechanics, College of Engineering,
Virginia Polytechnic Institute and State University, USA
E-mail: haref@vt.edu

One of the most spectacular, significant and well studied fluid instabilities is the formation of a vortex street wake behind a bluff body. Von Kármán was the first to present a quantitative model of the "vortex street" wake as a double row of point vortices, to determine which configurations propagate in the direction of the rows, and to consider the linear stability theory for such states.

In the early literature on vortex streets one works with infinite rows of vortices. The vortex street is assumed to continue to infinity both upstream and downstream. Another analytical approach is to use periodic boundary conditions in the direction of the wake. This representation was used by Domm (1956) in his analysis of the instability of the Kármán vortex street. Birkhoff and Fisher (1959) were the first to treat vortices in a periodic strip as a dynamical system in its own right.

We have used the periodic system to address problems of vortex wake patterns, in particular vortex wakes that are more complicated than the traditional twovorticesperstrip configurations. We use the term "exotic wakes" for vortex wakes of this kind. We submit that this approach can yield a number of insights, including results of direct relevance to experiments, in the same sense that von Kármán's analysis has been helpful to the understanding of the regular vortex street wake, and we present some of the results obtained to date following this program. A number of key references for the talk are given below.

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MOTION OF RIGID BODIES AND VORTEX STRUCTURES IN A PERFECT FLUID

ALEXEY V. BORISOV, IVAN S. MAMAEV, SERGEY M. RAMODANOV

Institute of Computer Sciences, Russia
E-mail: borisov@ics.org.ru

We consider some problems of the rigid-body dynamics and the theory of point vortices. We start with the Chaplygin problem on the fall of a heavy rigid body in a perfect fluid. The circulation around the body is not necessarily zero. New integrable cases have been found. Moreover, in the general case, the system is shown to be non-integrable. Some results of numerical study of the chaotic behavior of the system are presented.

Then, we consider the system of a rigid body interacting with point vortices in an unbounded volume of ideal incompressible fluid. The case where the body is a circular cylinder is considered in detail. The system of a circular cylinder and a single point vortex is shown to be integrable. The system ceases to be integrable as soon as the body's shape is slightly perturbed: the system of an elliptic cylinder and a point vortex exhibits chaotic features. We also discuss the Hamiltonian form of the equations of motion for an arbitrary 2D body interacting with point vortices. In conclusion, to facilitate the numerical analysis of the system, we show how the equations can be simplified using various reduction procedures.

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ANALOGY OF A VORTEX-JET FILAMENT WITH THE KIRCHHOFF ELASTIC ROD AND ITS DYNAMICAL EXTENSION

YASUHIDE FUKUMOTO

Graduate School of Mathematics and Space Environment Research Center, Japan
E-mail: yasuhide@math.kyushu-u.ac.jp

The simplest asymptotic theory for self-induced motion of a vortex filament, embedded in an incompressible fluid, is the localized induction approximation (LIA) which gives rise to a completely integrable evolution equation. Behind this integrability lies the localized induction hierarchy (LIH), an infinite sequence of the commuting Hamiltonian vector fields [1]. Moore and Saffman [2] derived an evolution equation of a vortex filament with axial velocity in the core, or a vortex-jet filament. In the spirit of the LIA, the Moore-Saffman equation is reduced to an evolution equation of the centerline X(s, t), as functions of the arclength s and the time t, of a filament carrying the circulation Γ in the form [3]:

$$\frac{\partial \boldsymbol{X}}{\partial t} = \Lambda \boldsymbol{X}_s \times \boldsymbol{X}_{ss} + W\left[\boldsymbol{X}_{sss} + \frac{3}{2}(\boldsymbol{X}_{ss}\cdot\boldsymbol{X}_{ss})\boldsymbol{X}_s\right]; \quad \Lambda = \frac{\Gamma}{4\pi}\log\left(\frac{L}{\sigma}\right), \tag{1}$$

where L and σ are long and short-range cut-off parameters, W represents axial-flow flux, and Λ and W are taken as constants. The subscript s stands for differentiation with respect to s. The augmented term in (1) is no other than the second one of the LIH. Consider the permanent form of a vortex-jet filament translating as a whole with constant velocity $V\boldsymbol{e}_z$ with $\boldsymbol{e}_z$ being the unit vector in the z direction. We admit a slipping motion of the filament along itself, with velocity c_0 , and thus the filament obeys $\boldsymbol{X}_t = -c_0\boldsymbol{t} + V\boldsymbol{e}_z$, with $\boldsymbol{t}$ being the unit tangent vector. Upon substitution, (1) becomes

$$W\boldsymbol{t}_{ss} = V\boldsymbol{e}_z - (V\boldsymbol{e}_z\cdot\boldsymbol{t})\boldsymbol{t} - W(\boldsymbol{t}_s)^2\boldsymbol{t} + \Lambda\boldsymbol{t}_s\times\boldsymbol{t}. \tag{2}$$

If the parameter s is thought of as time, this is reckoned upon as an equation of a charged pendulum, of mass W, constrained to the surface of unit sphere S^2 [4]. The first two terms on the right-hand side of (2) represents the gravity force $V\boldsymbol{e}_z$, and the third term is the centrifugal force. The last term is interpreted as the Lorentz force with the external magnetic field generated by a magnetic monopole sitting at the sphere center: $\Lambda\boldsymbol{t}/|\boldsymbol{t}|^3$.

The above charged spherical pendulum is a completely integrable Hamiltonian system, which is identical with the heavy symmetrical top, fixed at a point [4] and the equilibrium shape of a thin elastic rod of circular cross-section. The latter, being an extension of Euler's elastica to three dimensions, is called the Kirchhoff elastic rod. The axial-flow flux parameter W plays the role of the bending stiffness, and Λ is related to the torsional rigidity. This analogy is different from the known one between the Kida class [5] of the LIA equation and Kirchhoff's elastica [6]. The present analogy provides more direct correspondence in the sense that not only the bending but also the torsional deformation of an elastic rod finds its correspondent in (1).

The analogy encompasses some dynamics of an thin extensible elastic rod of circular cross section, of area S, uniform along the rod. We ignore the external body force. Let $\boldsymbol{r}(\xi,t)$ be the centerline of a rod and $\boldsymbol{\Omega}(\xi,t)$ and $\boldsymbol{\omega}(\xi,t)$ be the rate of rotation of the coordinate frames along the rod and with respect to time. If the internal stress $N(\xi,t)$ is acted at each cross-section, the dynamical equations for the balance of the linear and angular momenta read

$$\rho_0 S\frac{\partial^2\boldsymbol{r}}{\partial t^2} = \frac{\partial}{\partial\xi}\boldsymbol{N}: \quad \frac{\rho_0}{\sqrt{g}}\frac{\partial}{\partial t}\left(I\boldsymbol{t}\times\frac{\partial\boldsymbol{t}}{\partial t} + 2I\omega_3\boldsymbol{t}\right) = \frac{1}{\sqrt{g}}\frac{\partial}{\partial\xi}\left(A\boldsymbol{t}\times\frac{\partial\boldsymbol{t}}{\partial\xi} + C\Omega_3\boldsymbol{t}\right) + \boldsymbol{t}\times\boldsymbol{N}, \tag{3}$$

where ρ_0 is mass density, per unit length, in equilibrium, I is inertia moment, A and C are the bending stiffness and the torsional rigidity respectively, and $\omega_3 = \boldsymbol{\omega}\cdot\boldsymbol{t}$, $\Omega_3 = \boldsymbol{\Omega}\cdot\boldsymbol{t}$.

Allowance is made for extension or contraction of the rod ($g = |\partial \boldsymbol{r}/\partial\xi| \neq 1$). A whole family of permanent form of an extensible rod of circular cross-section propagating with a constant speed is constructed. The traveling speed is determined, via the first of (3), by the elastic force with constant rate of stretching. The shape is determined by the second one. Comparison of (3) with its static version reveals that the traveling wave take the same form as Kirchhoff's elastica, and thus as the permanent form of a vortexjet filament. We establish a variational principle for dynamics of an elastic rod, with respect to the tangential vector and the perpendicular directors along the centerline, or material frames, which yield not only the equations for balance of linear and angular momenta but also torsional vibration equation. The origin of the analogy with the vortex-jet filament is manifested by a reduction of the torsional energy with the help the rotational symmetry about the rod axis.

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FORCED AND DECAYING 2D TURBULENCE

GERTJAN VAN HEIJST

Fluid Dynamics Laboratory, Department of Physics, Eindhoven University of Technology, The Netherlands

E-mail: g.j.f.v.heijst@tue.nl

The inverse energy cascade present in two-dimensional (2D) turbulence leads to the formation of large-scale flow structures. In the case of decaying 2D turbulence on a square domain with no-slip walls the flow usually shows self-organization into a single domain-filling circulation cell with an associated increase in the total angular momentum of the flow - a process referred to as 'spontaneous spin-up'. Subsequently, this organized state may persist until all energy is depleted by viscous dissipation and the fluid eventually comes to rest. In contrast, if the energy of the flow is maintained by some external forcing mechanism, a spectacularly different behaviour may be observed. Boundary layers present at the domain walls can destabilize the organized state, such that the dominating circulation cell collapses, and the self-organization process may start anew. Most strikingly, the circulation may even show sign reversal. This flow behaviour has been investigated by high-resolution numerical simulations based on spectral techniques.

THREE-DIMENSIONAL DOUBLY - PERIODIC TRAVELLING GRAVITY WATER WAVES

GÉRARD IOOSS[1], PAVEL I. PLOTNIKOV[2]

[1]IUF, INLN UMR 6618 CNRS - UNSA, France

[2]Lavrentyev Institute of Hydrodynamics, Siberian Division of Russian Academy of Sciences, Russia *E-mail: gerard.iooss@inln.cnrs.fr*,

We consider doubly-periodic travelling waves at the surface of an infinitely deep perfect fluid, only subjected to gravity g and resulting from the nonlinear interaction of two simply periodic travelling waves making an angle 2θ between them.

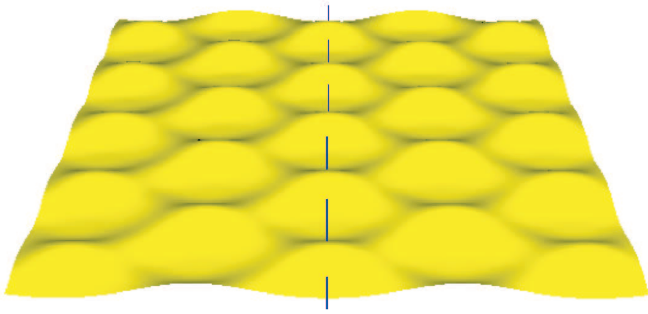

Fig 1. 3-dim travelling wave, $\theta = 26.5^o$. The dashed line is the direction of propagation of the waves. Crests are dark and troughs are grey.

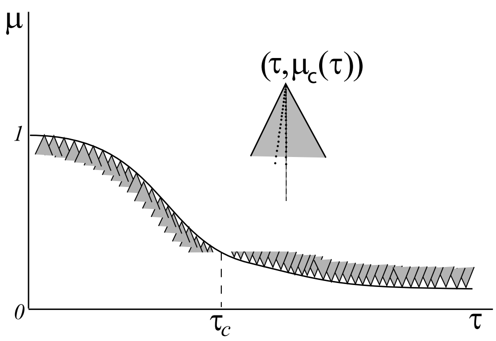

Fig 2. Small sectors where 3-dimensional waves bifurcate. Their vertices lie on the critical curve $\mu = = \mu_c(\tau), \tau = \tan\theta$. The good set of points is asymptotically of full measure at the vertex on each half line (see the detail above).

Denoting by $\mu = gL/c^2$ the dimensionless bifurcation parameter (L is the wave length along the direction of the travelling wave and c is the velocity of the wave), bifurcation occurs for $\mu = cos\theta$. For non-resonant cases, we first give a large family of formal three-dimensional gravity travelling waves, in the form of an expansion in powers of the amplitudes of two basic travelling waves. "Diamond waves" are a particular case of such waves, when they are symmetric with respect to the direction of propagation. The main object of the lecture is the proof of existence of such symmetric waves having the above mentioned asymptotic expansion. Due to the occurence of small divisors, the main difficulty is the inversion of the linearized operator at a non trivial point, for applying the Nash Moser theorem. This operator is the sum of a second order differentiation along a certain direction, and an integro-differential operator of first order, both depending periodically of coordinates. It is shown that for almost all the 3-dimensional travelling waves bifurcate for a set of "good" values of the bifurcation parameter having asymptotically a full measure near the bifurcation curve in the parameter plane (θ, μ).

UNSTABLE-PERIODIC-FLOW ANALYSIS OF COUETTE TURBULENCE

SHIGEO KIDA, TAKESHI WATANABE, TAKAO TAYA
Graduate School of Engineering, Kyoto University, Japan
E-mail: kida@mech.kyoto-u.ac.jp

Abstract

An unstable periodic flow (UPF), which represents the characteristics of a minimal Couette turbulence well, is applied for the study of stretching rate of passive line elements for the purpose of understanding the mixing mechanism in turbulence. It is shown that any two line elements, which start from a same position but with different directions in UPF, tend to align with each other in a few periods of UPF. This property of alignment guarantees that the direction (therefore the stretching rate as well) of line elements is uniquely determined by the spatiotemporal structure of UPF (perhaps of turbulence too). That is, the fields of direction and stretching rate of line elements can be uniquely defined for a given UPF, which enables us to directly compare the stretching rate of line elements and the instantaneous structure of flows. Here, the fields of various physical quantities associated with line elements are constructed by a long-term particle simulation. By taking the spatiotemporal correlation between these fields of line elements and the rate-of-strain field of UPF we find that there is a strong correlation between the stretching rate of line elements and the first eigenvector of the rate-of-strain tensor, and that not only the magnitude of the first eigenvalue but also the direction of the first eigenvector are relevant to the strong stretching of line elements.

Background

It is well-known that the turbulence state in Couette system is sustained if the Reynolds number $Re = Uh/\nu$, where U is the velocity of channel boundary, h is a half the channel width, and ν is the kinematic viscosity of fluid, is greater than a critical value ($\approx$ 320). In their direct numerical simulation of a minimal Couette system at $Re = 400$, Hamilton *et al.* (1995) discovered the regeneration cycle of streamwise vortices and the low-speed streaks. Recently, Kawahara & Kida (2001) found an unstable periodic flow which reproduces the above regeneration cycle as well as the mean velocity profile and spatial distribution of velocity fluctuations in turbulent state. Since the spatiotemporal structure is well-defined, the UPF is expected to be useful to investigate the typical dynamical characteristics of turbulence.

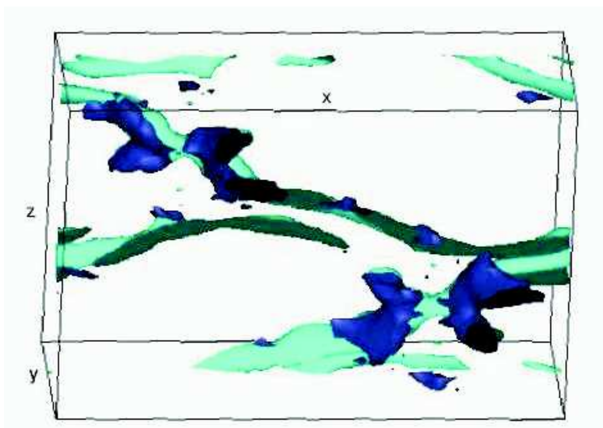

Fig 1. Strong stretching regions of passive line elements in an unstable periodic flow of the minimal Couette system. The regions of high values of stretching rate of line elements and of the first eigenvalue of the rate-of-strain tensor are shown respectively with iso-surfaces of dark grey and light grey.

Subject and Method

Strong mixing is one of the most important characteristics of turbulence. In order to clarify the mixing mechanism in turbulence we examine the deformation and stretching properties of passive line elements using the forementioned UPF.

We search for an unstable periodic motion in a minimal box which is different from and more appropriate for the present analysis than the one obtained before by Kawahara and Kida (2001). Since this is unstable to small disturbances, it cannot be realized by usual numerical simulation of fluid equations. Instead, it is captured by the Newton-Raphson iteration. Many particle pairs (regarded as line elements) are then submerged in the unstable periodic flow, and caclulated numerically the stretching rate and the direction of each pair over a long time period. By taking the statistical average of the position, the direction, and the stretching rate of particle pairs, we construct the fields of direction and stretching rate of passive line elements. Moreover, by taking the spatiotemporal correlation between the characteristic quantities associated with the line elements and the eigenvalues and the eigenvectors of the rate-of-straing tensor, we find that there are strong correlations between the stretching rate of line elements and the first eigenvector of the rate-of-strain tensor.

Results

The main relults are summarized in the above abstract. In figure 1 we compare the regions of strong stretching rate of line elements and high strain regions in a minimal Couette system. Observe that the two iso-surfaces agree fairly well with each other but not completely. This discrepancy is attributed to the deviation in directions between line elements and the first eigenvector of the rate-of-strain tensor. The detailed analysis of this and other dynamical properties will be discussed in the Symposium.

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MOTION OF AN ELLIPTIC VORTEX RING AND PARTICLE TRANSPORT

YOSHI KIMURA

Graduate School of Mathematics, Nagoya University, Japan
E-mail: kimura@math.nagoya-u.ac.jp

Transport of fluid particles by an isolated vortex has been a fundamental problem which, particularly if a vortex moves steadily, provides a direct example of long-surviving advection of materials from a turbulent region where a vortex is excited to a laminar region in fluids. The main objective of this paper is to demonstrate such transport by an elliptic vortex ring. Our starting point is that, as long as the ellipticity is small, the motion of an elliptic vortex ring is periodic in time, and it is well-described by the Local Induction Equation (LIE),

$$\frac{\partial \mathbf{X}}{\partial t} = \frac{\Gamma}{4\pi} \log\left(\frac{L}{\sigma}\right) \left[\frac{\partial \mathbf{X}}{\partial s} \times \frac{\partial^2 \mathbf{X}}{\partial s^2}\right] = G\kappa(s)\mathbf{b}(s), \tag{1}$$

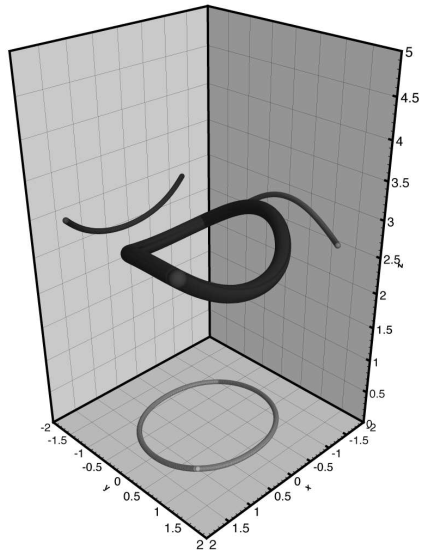

Fig 1. A snapshot of an elliptic vortex ring moving upward. The curve on each plane is a projection of the plane.

where $\mathbf{X}(s,t) = (x(s,t), y(s,t), z(s,t))$ is the position of the vortex segment parametrized by the arc-length s, at time t, and Γ is the circulation of the vortex ring. In the last equation, κ, and $\mathbf{b}$ are curvature and the binormal unit vector parametrized by s, and we assumed that the prefactor, $\frac{\Gamma}{4\pi}\log\left(\frac{L}{\sigma}\right)$, is a constant G which is called the self-induction constant. The usage of LIE for the motion of an elliptic vortex ring was first proposed by Arms and Hama [1]. Later Dhanak and de Bernardinis [2] argued that the LIE is an approximation which neglects such vortex instability as Crow instability or elliptic instability on the core.

Figure 1 shows a snapshot of an elliptic vortex ring moving upward. The initial aspect ratio $a/b = 1.5$, and the thickness of the core is exaggerated. The curve on each plane is a projection on to the plane. Similar figures are obtained also experimentally (Oshima *et. al.* [3]).

It is well-known that the LIE is equivalent to the Nonlinear Schrödinger equation (NLS)

$$i\frac{\partial\phi}{\partial t} + \frac{\partial^2\phi}{\partial s^2} + \frac{1}{2}|\phi|^2\phi = 0 \tag{2}$$

where

$$\phi(s,t) = \kappa(s,t)\exp\left[i\int^s \tau(s',t)ds'\right], \tag{3}$$

and the time has been rescaled by means of G (Hasimoto [4]). In the paper, the possibility of obtaining analytic solutions for the motion of an elliptic vortex ring by means of the algebro-geometric method (finite gap solutions) will be discussed (Grinevich and Schmidt [5], Calini and Ivey [6]).

For the motion of particles, the Biot-Savart integral,

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{\mathbf{X}'(s,t)\times(\mathbf{r}-\mathbf{X}(s,t))}{|\mathbf{r}-\mathbf{X}(s,t)|^3}\mathrm{d}s \tag{4}$$

is used to calculate the induced velocity at the position of a particle $\mathbf{r}(t)$. For simplicity, we set the circulation $\Gamma = 1$, which means that a relation $\log\left(\frac{L}{\sigma}\right) = 4\pi G$ is imposed for the thickness (i.e. the ratio between the long and short length scales) of the core.

As a fundamental transport property, the fluid volume carried by a vortex ring is of interest. This quantity could be approximately measured by the number of particles trapped by the vortex ring. Unfortunately, however, even if the analytical solutions for the motion of an elliptic vortex ring are known, it may be difficult to calculate the exact amount of the volume. The situation would be analogously explained by the corresponding 2D problem of a perturbed vortex pair (Rom-Kedar, Leonard and Wiggins [7]). In this situation, the unstable manifold of a vortex pair (i.e. the boundary of the pair) is dramatically deformed by time periodic sinusoidal perturbations to produce many lobes outside. For the present 3D case, if the aspect ratio is small, we may construct a problem of a perturbed circular vortex ring. Numerically, we can observe scattered particles in the wake of an elliptic vortex ring. Those scattered particles may indicate the existence of similar lobes in this 3D perturbation problem also.

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STABILITY OF HETONIC QUARTETS. EXPLORING TRANSITIONS IN BAROCLINIC MODONS

ZIV KIZNER

Departments of Physics and Mathematics, Bar-Ilan University, Israel
E-mail: zinovyk@mail.biu.ac.il

A new concept, a hetonic quartet, is presented. A hetonic quartet is a two-layer quasigeostrophic anti-symmetric ensemble of four synchronously translating discrete vortices aligned perpendicularly to the axis of their translation and anti-symmetry. Such a discrete construction shares some traits with baroclinic modons and, therefore, offers a relatively simple, finite-dimensional model for study of the transitions observed in baroclinic modons [6]. The beta-plane baroclinic modon with a circular frontier (separatrix) is a superposition of an anti-symmetric barotropic modon (i.e., a distributed barotropic vortex pair) and a circularly symmetric baroclinic component (rider). Its steady zonal translation is due to the self-propulsion of the barotropic vortex pair only. The baroclinic mode (rider) has no effect on the modon's translation in this case and, therefore, its amplitude can be arbitrary [1]. Smooth baroclinic modons [4, 5] are marked by continuity of their vorticity fields. A smooth circular baroclinic modon appears as two chunks of vorticity of opposite signs, which reside at different depths (one in the upper layers and the other in the lower layers) and are shifted relative to each other in the north-south direction. This solution was recently shown to be a particular and degenerate case of smooth baroclinic modons with elliptical separatrices [7]. In a smooth elliptical modon, baroclinic and barotropic modes are interdependent, the amplitude of the baroclinic mode is not arbitrary and the modon's eastward translation is associated with both modes. Therefore, the baroclinic mode should not be referred to as a rider in this case. When ellipses are sufficiently extended meridionally, the barotropic and baroclinic modes are close in magnitude, and such modons behave stably in numerical simulations [7]. With increasing separatrix aspect ratio, the northern (positive) and southern (negative) vortices become more and more localized (i.e., their vorticity peaks rise), while their overlap decreases and becomes negligible. These characteristics demonstrate that an elliptical modon is affiliated with a heton, a pair of rigidly coupled discrete vortices of the same strength but opposite signs that are confined to different layers of a two-layer fluid [2, 3]. In numerical simulations, two- and three-layer smooth circular modons with riders of moderate strength behave quite stably. However, when exposed to small perturbations over prolonged periods, such modons eventually make transitions to even more stable, heton-like elliptical modon states, in which the two main vorticity chunks hardly overlap [6]. The transition starts with an oscillation, whose period is much longer than that of the forcing perturbation, and is a manifestation of intrinsic processes induced by the forcing and related to the loss of stability. In the elliptical state attained, the translation speed and the separation between vorticity peaks of the upper and lower vortices are grater than those in the initial circular state. The above observations raise a number of questions. Why do considerably extended elliptical modons remain essentially insensitive to the prolonged action of small perturbations? Why does, in contrast, an apparently stable circular modon subjected to the same perturbations drastically change its parameters, in particular, its separatrix form, the separation between its vorticity peaks, and its translation speed? Is the increase in separation and translation speed (with the subsequent stabilization) the only feasible development, or are other outcomes, such as rapprochement and/or deceleration, possible? Is the overlap of the main upper and lower vortices (in the circular modon) favorable to the transition? Is the beta- effect in the second transition as significant as in the first transition? The answers to these questions are searched for by employment of the hetonic quartet concept. On the f-plane, a hetonic quartet is made up of conventional point vortices (confined to either the upper or the lower layer) with specially fitted circulations and distances between each other. On the beta-plane, a two-layer analog of the barotropic modulated point vortex model [8] is applied. A necessary and sufficient condition for the stability of f-plane hetonic quartets to anti-symmetric perturbations is established using the analytical methods of Hamiltonian dynamics, while on the beta-plane, a necessary (linear) stability condition is determined. As distinct from hetons, a stable hetonic quartet is not rigid: when slightly moved off equilibrium it undergoes elastic oscillations. Continuously or periodically acting small anti-symmetric perturbations force a stable hetonic quartet to split up into two pairs of hetons, each traveling at different speeds along the same axis. The separation in the faster heton is generally greater than that between the 'centers of mass' of the upper- and lower-layer vortices of the original hetonic quartet. Beta-effect is not a deciding factor in this transition. The similarity between baroclinic modons and hetonic quartets is traced, and, based on the results obtained with hetonic quartets, the following scenario of transitions in baroclinic modons is put forward. The circular modon state with a moderate rider is presumably stable, but is located close to the stability margin in the parameter space. Small periodical or continuously acting perturbations can force the modon out of the stability region and make it unsteady. An unsteady modon sheds some vorticity or just breaks up into a couple (or a chain) of vortical structures traveling at different speeds, the strongest of them — the eastward-going elliptical modon — being stable. Because of its relatively simple heton-like structure (i.e., because of the lack of overlap), this modon is more resistant to perturbations than the circular modon is, and therefore, it survives in long-term simulations.

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ON LAGRANGIAN TURBULENCE

VALERY V. KOZLOV

V. A. Steklov Mathematical Institute, Russian Academy of Sciences, Russia
E-mail: kozlov@pran.ru

By the Lagrangian turbulence is meant a chaotic advection (transport) of liquid particles. In my talk, I will present some rigorous results on stochastic behavior of dynamical systems governing the motion of particles.

In particular, existence conditions for first integrals (conservation laws), symmetry fields and integral invariants will be indicated. It turns out that typical stationary velocity fields of viscous liquids do not admit non-trivial tensor invariants. For a stationary flow of an ideal liquid to be stochastic, the velocity field and its curl must be collinear. Plus, some issues related to chaotization of motion of liquid particles caused by a periodic perturbation of the velocity field will be discussed.

STABILITY AND PHASE TRANSITIONS TO SUPER-ROTATION IN BARATROPIC VORTEX DYNAMICS ON A ROTATING SPHERE

CHJAN C. LIM

Mathematics Department, Rensselaer Polytechnic Institute, USA
E-mail: limc@rpi.edu

Starting with Kraichnan's 1975 Gaussian theory of non-rotating 2D vortex dynamics, I will review new results that limit the range of validity of our recent extensions of the Gaussian model to barotropic vortex dynamics on first the non-rotating sphere and then the rotating sphere. Exact partition functions for Kraichnan's theory will be presented where they are valid. This part of the talk provides a much needed update and extensions of the Onsager-Kraichnan theory of 2D statistical hydrodynamics to rotating barotropic vortex dynamics.

It is clear that the next step in any attempt to advance the Onsager-Kraichnan theory of the statistical mechanics of rotating 2D flows is to introduce a model which is valid in the full range of inverse temperatures $\beta \in (-\infty, \infty)$ since phase transitions could in principle occur at any temperature. This step is based on Kac's spherical model where the relative enstrophy constraint is now enforced in a microcanonical way. I will review an exact partition function of the spherical model for barotropic vortex dynamics on the non-rotating sphere and show that this model is not exactly integrable on the rotating sphere.

The next steps to advance the statistical mechanics of barotropic flows on a rotating sphere is two-fold. One, I introduce a simple mean field theory which supports a rich critical phenomenology in terms of the invariants in the model, namely, the rate of spin of the sphere and the relative enstrophy and total kinetic energy of the barotropic flow. Theorems based on free energy calculations of the simple mean field theory will be presented, stating the positive and negative critical temperatures of statistical barotropic dynamics in terms of physical parameters in the rotating problem.

Two, I derive a family of finite dimensional spin-lattice models which converges to the spherical model for barotropic flows on a rotating planet, and simulate the phase transitions using Monte-Carlo methods. This family of spin-lattice Hamiltonians models the total kinetic energy of rotating barotropic flows on a sphere in terms of a piecewise constant relative vorticity distribution obtained from a Voronoi mesh. The critical universality class of this family is anti-ferromagnetic Ising type with global logarithmic interactions. It supports a negative critical temperature for any rate of spin between a disordered phase and a super-rotating solid-body phase at extremely high energy (equivalently, extremely hot near 0 negative temperatures); and for large enough planetary spins, it supports a positive critical temperature between a disordered phase and a counter-rotating solid-body phase at extremely low energy (or equivalently, extremely cold, near 0 positive temperatures).

I will give detailed comparisons between Monte-Carlo results on the critical phenomenology of the spherical model and exact results from the mean field theory in this talk. The agreement between numerical results, simple mean field theory and exact partition function results is remarkable. They all predict a marked asymmetry for the critical phenomenology of barotropic vortex dynamics on a sphere, between positive and negative temperatures and likewise, between pro and counter-rotating statistical equilibrium states in the case of the rotating sphere. This talk summarizes the considerable effort in the past five years to model the global and statistical balances between the kinetic energy, enstrophy and angular momentum of a rotating single-layer (divergence free) atmosphere in a way that is amenable to the methods of modern applied and computational mathematics. Unlike the huge amounts of time and money spent on simulating the GCM which models complex torques between the multi-layered atmosphere and solid planet, my approach is based on the minimalist programme of investigating whether the smallest possible geophysically relevant models are capable of supporting atmospheric phenomena pertinent to global warming and climate change such as atmospheric super-rotation and wobble. I believe that key aspects of our results will be relevant to the simulations of the full GCM. Atmospheric scientists should be open to the considerable benefits of accepting this possibility.

Future work in this field will include extensions of statistical mechanics and variational analysis to the Shallow Water Equations on a rotating sphere (divergent flows) and to two-layer baroclinic models on a sphere. The numerical results in this talk are based partly on the joint work of the author with Xueru Ding and supported by the ARO and DOE.

NEAR WALL CONTROL AND TRANSPORT PROPERTIES OF TWO-DIMENSIONAL VORTEX STRUCTURES

VYATCHESLAV V. MELESHKO

Department of Theoretical and Applied Mechanics,
Kiev National Taras Shevchenko University, Ukraine
E-mail: meleshko@univ.kiev.ua

The talk presents the results of an investigation of the formation, stability and control of the localized two-dimensional vortex structures near a curved rigid airfoil with respect to the drug reduction and the transport properties of passive admixtures. The low order models based upon the classical discrete vortices imbedded into an inviscid fluid are employed. The unsteady flow near the airfoil is modeled by a nascent point vortex of variable intensity satisfying the Kutta-Joukovskii condition during transient processes. Analytical and numerical studies of the problems of vortex interactions with microgrooves and cavities (vortex chambers) are presented.

Special attention is paid to the stationary equilibrium positions of vortices near a rigid body. Recommendations concerning optimal ways of trapped vortex control are given. In particular, the recommendations for optimal choice of positioning and intensity of vortex generators on the controlled surface are given.

Numerical results of simulations of the transport processes in the near-wall zone, the determination of regions of low and high domains of passive admixtures are discussed. A special technique for the quantitative estimate of the quality of stirring properties is given.

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VORTEX STRUCTURES: THE LEGACY OF HELMHOLTZ AND KELVIN

KEITH MOFFATT

University of Cambridge, Department of Applied Mathematics
and Theoretical Physics Centre for Mathematical Sciences, UK
E-mail: h.k.moffatt@damtp.cam.ac.uk

The year 2007 will mark the centenary of the death of William Thomson (Lord Kelvin), one of the great nineteenth-century pioneers of vortex dynamics. Kelvin was inspired by H. Helmholtz's (1858) famous paper "Ueber Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen", translated by P. G. Tait and published in English (1867) under the title "On Integrals of the Hydrodynamical Equations, which express Vortex-motion".

Kelvin conceived his 'Vortex theory of Atoms' (1867-1875) on the basis that, since vortex lines are frozen in the flow of an ideal fluid, their topology should be invariant.

We now know that this invariance is encapsulated in the conservation of helicity in suitably defined Lagrangian fluid subdomains. Kelvin's efforts were thwarted by the realisation that all but the very simplest three-dimensional vortex structures are dynamically unstable, and his vortex theory of atoms perished in consequence before the dawn of the twentieth century.

The course of scientific history might have been very different if Kelvin had formulated his theory in terms of magnetic flux tubes in a perfectly conducting fluid, instead of vortex tubes in an ideal fluid; for in this case, stable knotted structures, of just the kind that Kelvin envisaged, do exist, and their spectrum of characteristic frequencies can be readily defined.

This introductory lecture will review some aspects of these seminal contributions of Helmholtz and Kelvin, in the light of current knowledge.

ON ADIABATIC INVARIANCE IN VOLUME-PRESERVING SYSTEMS

ANATOLY NEISHTADT

Space Research Institute, Russian Academy of Sciences, Russia
E-mail: aneishta@iki.rssi.ru

We conside 3D and 4D volume-preserving systems which differ by a small perturbations from integrable systems. The talk contains review on two problems: passages through separatrix in 3D systems and passages through resonances in 4D systems.

In 3D case we assume that in the phase space of the unperturbed system there are domains filled with closed trajectories, and these domains are separated by two-dimensional separatrices. Under the action of perturbation phase points cross these separatrices. For motion far from separatrices the system has an adiabatic invariant (approximate first integral, which is a first integral of averaged over unperturbed dynamics system). When the phase point crosses a narrow neighborhood of separatrices the value of adiabatic invariant undergoes a quasi-random jump. We discuss asymptotic formulas for this jump in various situations. For multiple separatrix crossings presence of these jumps lead to destruction of adiabatic invariance and chaotic behavior in the system.

In 4D case we assume that the phase space of the unperturbed system is foliated into two-dimensional invariant tori. Averaging over unperturbed motion is used for approximate description of the perturbed dynamics. The averaged system has a first integral, which is an approximate first integral of the exact system, i.e. an adiabatic invariant. Resonant phenomena (capture into resonance, scattering on resonance) lead to inapplicability of averaging, destruction of adiabatic invariance, dynamical chaos and transport in large domains in the phase space. We present perturbation theory methods for description of capture into resonance and scattering on resonance.

THE VALIDITY OF THE NONDIVIRGENCE ASSUMPTION IN 2D VORTEX DYNAMICS ON THE ROTATING EARTH

NATHAN PALDOR

Institute of Earth Sciences, The Hebrew University of Jerusalem, Israel
Email: nathan.paldor@huji.ac.il

The dynamics of fluids in the ocean or the atmosphere of the rotating planet Earth is described by the Shallow Water Equation (SWE), which represent the application of Euler equations to a rotating, inviscid and incompressible fluid that occupies a thin layer compared to Earth's radius. No solutions are known for the nonlinear partial differential equations and, for the most part, solutions are obtained by studying the vorticity conservation equation, which greatly simplifies the three SWE to a single equation. The use of vorticity dynamics yields several types of known solutions such as linear waves, growing perturbations (i.e. instabilities) and integral constraints. However, formally, the vorticity equation in 2D is an equivalent descriptor to the SWE only for nondivergent flows, i.e. flows contained in a layer of fluid that is bounded from above by a "rigid lid".

As I will show in my talk, in the presence of rotation an initial nondivergent flow becomes divergent at some later time, which limits the applicability of the nondivergence assumption to short times only. The generation of divergence by the vorticity has its origin in the presence of Coriolis force and is similar in nature to the generation of one horizontal velocity component starting from the second one by the same force. When the Coriolis frequency is not uniform in space (e.g. when it is expanded in power series of the latitudinal distance from mean latitude) the temporal changes in divergence become even more pronounced so the nondivergence assumption is invalidated at shorter times. The general considerations regarding the consistence of the nondivergence assumption have implications regarding the applicability of results obtained from the vorticity equation to the corresponding results of the SWE. Two such examples will be presented in some details: the characteristics of linear propagating wave solutions in the two systems and shear flows instability. For linear waves it will be shown that the nondivergent assumption greatly limits the wavenumber range for which a given propagating wave is an acceptable solution. For the classical shear flow example of a cos2 jet in a channel it will be demonstrated that removing the "rigid lid" and allowing the fluid to diverge actually stabilizes the, otherwise unstable, flow. This stabilization occurs despite the fact that in the divergent case there are additional types of propagating wave solutions compared to the nondivergent case, which should have destabilized the flow by allowing more room for the real waves to coalesce. Some of the possible reasons for this stabilization rather than destabilization of the nondivergent instabilities by allowing the 2D flow to diverge are related to the structure of the singular point where the wavelike perturbation is stagnant relative to mean flow (i.e. the point where the mean flow and the wave's phase speed are equal).

NONLINEAR DYNAMICS OF SEMI-TRANSPARENT EQUATORIAL WAVEGUIDE

GRIGORY M. REZNIK[1], VLADIMIR ZEITLIN[2]

[1]P. P. Shirshov Institute of Oceanology, Russian Academy of Sciences, Russia
[2]Laboratoire de Météorologie Dynamique CNRS - ENS, France
E-mail:greznik11@yahoo.com

Joint influence of the Earth rotation and sphericity results in existence of a special narrow domain in the vicinity of the Earth equator in the ocean and atmosphere- the so-called equatorial waveguide. This waveguide is semi-transparent in the sense that together with trapped equatorial baroclinic wave modes the non-trapped barotropic Rossby waves exist, which, in the linear approximation, propagate freely across the equator. We study non-linear interaction between the trapped equatorial baroclinic Rossby waves or/and Yanai waves and the free non-trapped barotropic Rossby waves using the model of two-layer rotating shallow water on the equatorial beta-plane.

Given non-trapped Rossby wave can resonantly interact with a pair of trapped Rossby or Yanai modes, or one Rossby and one Yanai mode; the ensemble of such pairs is determined by the parameters of the barotropic wave. The trapped modes in the triad can be identical, when the time and space periods of the baroclinic wave are twice the corresponding periods of the barotropic wave (parametric resonance case). If the frequencies of the trapped waves in such triad are both lower than the frequency of the non-trapped wave, the interaction results in exponential growth of the trapped mode amplitudes. In turn, the growing trapped modes generate a secondary barotropic mode having the form of reflected and transmitted wave structures spreading out of the equator. The interaction between the secondary barotropic and the trapped modes arrests the growth of these latter. The trapped mode amplitudes are gradually saturated obeying Landau type equations; the saturation level largely exceeds the initial amplitudes. The saturation, however, is stationary only in the parametric resonance case; when the two different baroclinic modes are excited their saturated amplitudes continue to oscillate slowly with frequencies depending on the constant amplitudes moduli. A general tendency observed by calculating typical increments and saturation levels is that two waves saturate at considerably different levels, the saturation level of longer wave being larger.

When the effects of spatial modulation are taken into account, a Ginzburg-Landau type equation describes the envelope of the excited trapped wave in the parametric resonance case. Numerical solution of this equation exhibits characteristic domain-wall like phase defects and "dark soliton" structures. In the general case of a pair of waves a hyperbolic system of coupled nonlinear equations arises, with the behaviour predicted by the non-modulated equations modified by possible shock formation.

We believe that the analogous results are valid for semi-transparent waveguides supporting dispersive waves whatever their nature. In the geophysical fluid dynamics context the results provide a mechanism for mid-latitudes - low-latitudes teleconnections via the barotropic Rossby waves, and give some hints on the dynamical origin of slow oscillations in the equatorial region.

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VIBRODYNAMICS OF SOLID+FLUID

VLADIMIR A. VLADIMIROV

University of York, UK
E-mail: vv500@york.ac.uk

The paper is devoted to the studying of a dynamical system ‘solid+fluid’ in the presence of vibrations using the double-timing averaging method. First, we consider the celebrated example of the Stephenson - Kapitza pendulum using a new version of the averaged method and its least action formulation. We directly exploit the least action principle, in which an averaging procedure appears most naturally and conservation laws follow automatically; its main advantage is a substantial decrease of the required amount of analytical calculations that are typically cumbersome for the all averaging method. Then, we consider the dynamics of a rigid sphere in an inviscid incompressible fluid, which completely fills a vibrating vessel of an arbitrary shape. The sphere can be either homogeneous or inhomogeneous in density. The results provide a full model for the averaged (or ‘slow’) motions, including the ‘slow Lagrangians’, the ‘slow potential energy’, and the ‘vibrogenic’ force, exerted by a surrounding fluid on a solid. We outline our calculations, present results in general forms, and discuss related examples, properties, and conjectures. A detailed consideration is given to the dynamics of a sphere with a ‘shifted’ centre of gravity. In the case of an infinite fluid it is mathematically equivalent to the Stephenson - Kapitza pendulum, while in a finite domain its behaviour is more complex. Attention is also paid to the justification of the method used, including the explicit calculation of the error.

HIGH DIMENSIONAL HAMILTONIAN DYNAMICAL SYSTEMS: THEORY AND COMPUTATIONAL REALIZATION FOR THEORETICAL CHEMISTRY

STEPHEN WIGGINS

School of Mathematics, University of Bristol, UK
E-mail: s.wiggins@bristol.ac.uk

In the early development of applied dynamical systems theory it was hoped that the complexity exhibited by low dimensional nonlinear systems might somehow lead to ways of understanding the complex dynamics of high dimensional systems. Unfortunately, there has not be great progress in this area. The most fruitful approaches to high dimensional systems involve deriving a (hopefully rigorous) reduction to a low dimensional system.

Nevertheless, applications cannot wait for theory, and the availability of high performance computing resources has led to a great deal of computational studies of high dimensional systems.

But even under these circumstances, the problem of high dimensionality often forces one to make severe assumptions on the dynamics in order to derive physically relevant quantities from the model, e.g., ergodicity assumptions may be necessary in order to deduce a reaction rate from a computation.

We approach the problem of high dimensionality from the other direction. Our interest is in the exact Hamiltonian dynamics of high dimensional systems. In recent years a combination of theoretical and computational advances have provided a new class of tools for certain types of high dimensional problems. In particular, the problems we discuss here are related to molecular dynamics and reaction rates. We are concerned with the dynamical mechanisms leading to reaction.

As applied dynamical systems theory developed and expanded throughout the 70's and 80's there was much effort in applying these global, geometrical concepts and techniques to problems related to the dynamics of molecules. In the early 90's this effort began to die out in the chemistry community because the approach did not appear to apply to problems with more than two degrees-of-freedom. New concepts were required. In the past few years there has been much progress along these lines. Some of the necessary concepts already existed in the dynamical systems community, but computational algorithms enabling their application to concrete problems were not available. We will discuss these recent developments and their application to the understanding of a variety of issues related to the dynamics of molecules.

Theoretically, we have constructed a dynamically exact phase space transition state theory, for which we can rigorously construct a "surface of (locally) no return" through which all reacting trajectories must pass. It can also be shown that the flux across the surface we construct is minimal. Central to this construction is a normally hyperbolic invariant manifold whose stable and unstable manifolds enclose the phase space conduits of all reacting trajectories. They enable us to determine the volume of trajectories that can escape from a potential well (the "reactive volume"), which is a central quantity in any reaction rate.

The application of these ideas to concrete problems relies on the computational realisation of these structures. These can be realized locally through the Poincare-Birkhoff normal form, and then globalised. Recent advances in computational techniques enable one to carry out this procedure for systems with a large number of degrees of freedom. A similar set of techniques can be developed to deal with the corresponding quantum mechanical system. In particular a quantum normal form is used to determine quantum mechanical resonances and reaction rates with high precision.

In this talk we describe the theory, applications, and computations that make this possible. We will use HCN isomerization and the Muller-Brown potential to illustrate the ideas and methods and point out a number of areas where more close collaborations between chemists and applied mathematicians could prove fruitful.

This is joint work with A. Burbanks, H. Waalkens, R. Schubert.

CONVECTIVE TURNOVERS

VICTOR I. YUDOVICH

Department of Mechanics and Mathematics,
Rostov University, Russia
E-mail: yudovich@math.rsu.ru

We develop the asymptotic theory of Oberbeck-Boussinesq convection for the case of small heat conductivity and large viscosity. The corresponding regimes look like periodic or chaotic sequences of convective turnovers. The time between two turnovers is very long, of inverse heat conductivity order of magnitude.

DYNAMICS OF ALMOST PARALLEL VORTEX LINES

VLADIMIR E. ZAKHAROV

Landau Institute for Theoretical Physics, Russian Academy of Sciences, Russia
University of Arizona, USA
Waves and Solitons, LLC, USA
E-mail: zakharov@math.arizona.edu

From geometrical view-point the ideal-fluid is a Poisson system. The phase space is foliated to symplectic leaves. Each leaf is supplied with the Hamiltonian structure, which is defined by initial data. In a general case, this structure can be very complicated. In this talk we shell discuss only the simpliest possible symplectic leaf-system of vortex tube, homeomorphic to the set of parallel straight lines. In this case the Euler equations can be reduced to one equation for the complex function

$$\Psi = x(z, \vec{s}, t) + iy(x, \vec{s}, t). \tag{1}$$

Here $\vec{s}$ is two-dimensional "marker", z is a vertical coordinate.

The vortex configuration is described by the Casimir function $\Omega(\vec{s})$ defined by initial data. Function Ψ obeys the complicated integro-differential equations which could be simplified if the vortex lines are almost parallel. Systematic expansion in powers of small parameter

$$\varepsilon = \left|\frac{d\Psi}{dz}\right|$$

leads to a system of coupled nonlinear Shrödinger equations (NLSE) with exotic nonlinearity. The simpliest case of two parallel (antiparallel) vorticies of equal intensity is described by the following equation

$$i\Psi_t + \Psi_{zz} \pm \frac{\Psi}{|\Psi|^2} = 0. \tag{2}$$

This system has a family of self-similar solutions

$$\Psi = (t_0 - t)^{1/2 + i\alpha} F\left(\frac{z}{(t_0 - t)^{1/2}}\right) \tag{3}$$

describing formation of singularity in a finite time. For antiparallel vorticies this is reconnection. Infinite systems of NLSE, describing more complicated vortex configuration also have self-similar solutions. The question of applicability of the self-similar solution near the collapse point is discussed.